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MARCH, 1921

A Wide-Angle Astronomical Doublet 123
FRANK E. ROSS

The Present Status of the Constants and Verification of the Laws of Thermal Radiation of a Uniformly Heated Enclosure 131
W. W. COBLENTZ

Spectroradiometric Studies of Polarization Phenomena 156
L. R. INGERSOLL

Systems of Color Standards 160
A. AMES, JR.

The Orientation of the Grains in a Dried Photographic Emulsion 171
L. SILBERSTEIN

A Method of Obtaining Radiant Energy Having the Visible Spectral Distribution of a Complete Radiator at Very High Temperatures 178
IRWIN G. PRIEST

Courses in Optics and Optometry in Columbia University 184
JAMES P. C. SOUTHALL

The Angular Deflections Produced on Transmitted Light Rays by Slightly Incorrect Interfacial Angles of Reflecting Prisms 193
F. E. WRIGHT

The Spectral Distribution of Energy Required to Evoke the Gray Sensation 205
IRWIN G. PRIEST

Notices 210

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A WIDE-ANGLE ASTRONOMICAL DOUBLET.*

BY

FRANK E. ROSS.

For numerous special lines of work in astronomy there appears to be a need for photographic lenses having properties not to be found in existing lenses.

The degree of perfection of a lens or of an optical system is measured in terms of tolerances, absolute perfection being unattainable. The difficulty facing the astronomer in particular in the matter of lens equipment is due to his requiring smaller tolerances than those which govern the lens designer in the commercial development of the photographic lens. In general, the needs of the greatest number of photographic workers are met in a high-speed lens covering a wide field with moderate definition. The astronomer may or may not want a high-speed lens, but it is absolutely necessary that the definition be what is called critical from the centre to the edge of the plate. In general, the *central* definition of a well-made commercial lens, especially if retouched, is sufficiently good to meet the astronomer's exacting needs. But on passing outward from the axial or central region, the definition in the case of all lenses will "break" before many degrees have been passed over, defects of coma, astigmatism, and field curvature appearing, which are more or less serious, depending on the particular use to which the lens is put. In the case of several existing lens-types, these errors do not reach a serious amount until a point

* Communication No. 107 from the Research Laboratory of the Eastman Kodak Company.

5 degrees from the axis is reached. From 5 degrees to 20 degrees, or even to 30 degrees or more in some cases, the definition retains a certain average quality which is sufficiently good for the purpose for which the lens was designed. In short, the object of the designer has been to secure a large field of moderate definition rather than a small field of critical definition. So far as the writer is aware, in no case has a lens been designed which gives critical definition as much as 10 degrees from the axis, a flat photographic plate being presupposed.

The errors which arise in photographing an extended field are of two kinds; (1) errors in position; (2) errors in intensity, or photometric errors. Errors in position are due to general distortion of the image-field and to lack of symmetry in the image-cone where it cuts the plane of the photographic plate. This asymmetry is due to the defects of coma and astigmatism. Photometric field errors are due to asymmetry of image and field curvature. There is an additional photometric error due to vignetting by the mount of the lens, if the stop is not sufficiently small compared with the clear aperture, and to foreshortening of the stop itself. If the field of the lens is not flat, vignetting produces an error in position as well.

The lens about to be described is a by-product of the war-time activity of the Eastman Kodak Company in lens design and manufacture. It became necessary to design for the aerial service a lens of 6 inches aperture and 48 inches focal length, having a 15-degree field of absolute flatness. The field of this lens is shown in Fig. 2 (Hawk-Eye 48 inch). One of these lenses was constructed in 1918, and passed the most rigid test. An attempt to reduce the small outstanding astigmatism in this lens was successful to such an extent that it was felt that a lens had been designed which would meet the exacting needs of the astronomer in several lines of research where a wide field is of advantage. The aperture-ratio, or ratio of opening to focal length, is not as great as could be wished for certain lines of work, as, for example, in photographing nebulæ and comets. To what extent an extended field having perfect definition can be combined with a large aperture in a single lens can only be conjectured. The problem is one of enormous difficulty. It is, however, certain that the design of the present lens can be altered to produce a lens working at an aperture ratio as high as 1/7 without materially

affecting the definition over an extended field. But as there is a large program of work calling for an aperture ratio not greater than 1/14, it was deemed best to select this value as the limiting aperture, aiming with it as a foundation to produce a field as nearly perfect as possible. A further advantage in choosing the small opening is the greater thinness of the component lens-elements, giving a higher relative light-transmission and a shorter lens. It will be noted below that there are but four different radii, thus minimizing the labor of manufacture.

Specifications for Wide-Angle Astronomical Lens.

Glass (Catalogue of Chance Bros., January, 1920):

Cat. No.	μ_D	ν	μ_F	μ_G		
580	1.60650	58.0	1.61388	1.61984	Positives	A and D
458	1.54720	45.8	1.55568	1.56275	Negatives	B and C

The radii are, counting from the object side:

$R_1 = 47.70$ mm.; $R_2 = 186.639$ mm.; Biconvex Lens A
$R_3 = R_4 = R_5 = R_6 = 71.5925$ mm.; Biconcave Lens B and C
$R_7 = 186.639$ mm.; $R_8 = 45.62$ mm.; Biconvex Lens D

Thickness of lens elements:

$t_A = t_D = 2.3$ mm. $t_C = t_D = 1.0$ mm.

Separation of lens elements:

S_1 (axial distance between adjacent surfaces of A and B) $= 6.2$ mm.
S_2 (axial distance between adjacent surfaces of B and C) $= 5.9$ mm.
S_3 (axial distance between adjacent surfaces of C and D) $= 6.6$ mm.

Focal Length (G′ Line)	204.380 mm.
Back Focus (distance of focal point from rear surface)	193.674
Distance of first Gauss point within lens (G′ Line)	16.620
Distance of second Gauss point within lens (G′ Line)	10.706
Focal Length (A + B)	407.37 mm.
Focal Length (C + D)	310.13 mm.

Color correction:

Back Focus (F — G′ line) = +.015 mm.

A lens of any desired focal length can of course be constructed by multiplying all dimensions given by the appropriate factor.

Longitudinal Spherical Aberration and Sine Condition:

Aperture	Spherical Aberration mm.	Sine C mm.	Coma mm.
1/21	—.005	—.010	—.005
1/18	+.005	+.004	—.001
1/14	+.049	+.052	+.003

Field aberrations:

Angle	Distortion mm.	Secondary Image mm.	Primary Image mm.	Best Image mm.
5°	.0000	−.013	+.028	+.008
7½	−.0007	−.043	+.047	+.002
10	−.0024	−.025	+.018	−.004
12	−.0035	+.054	−.017	+.019
14	−.0057	+.209	−.157	+.026

The distortion, secondary and primary image in this table are for pencils pointed at the first Gauss point at the angles given. Let D be the distance at which any pencil, pointed at the first Gauss point at an angle α cuts the focal plane, measured from the intersection of focal plane and axis. If F is the focal length, the distortion in the above table is calculated from the formula

$$\text{Distortion} = D - F \tan \alpha.$$

The diameter of stop, placed centrally, giving a speed ratio of F/14 is 12.7 mm. The clear aperture of the lens elements is 20 mm. With a clear aperture of this amount there is no loss of intensity from the lens mount up to 10° from the axis. The loss of intensity from the foreshortening of the stop is small, of the order of 1½ per cent. at 10°.

Coma: Freedom from coma for small field angles is secured when the curves of spherical aberration and of sine condition are identical. From the values given above, this condition is seen to be fulfilled. For the larger angles, however, it is not always a true criterion for the absence of coma. It is accordingly necessary to trace trigonometrically a number of rays lying in the meridional plane parallel to each other and intersecting the stop at various distances from its centre. This has been done, with results given in the following table. The common inclination of the rays is 10°:

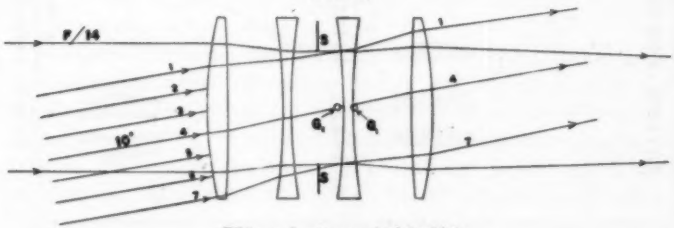
TABLE I.

Ray	D mm.	V		
1	36.037	7°	57′	22″
2	36.035	8	37	51
3	36.035	9	18	26
4	36.035	9	59	6
5	36.035	10	39	51
6	36.038	11	20	44
7	36.041	12	1	43

The rays and their course through the lens are shown to scale in Fig. 1. The calculations are for the G′ line. The central

ray, No. 4, is aimed at the first Gauss point. The rays are equally spaced, the perpendicular distance between consecutive rays being 2.457 mm. D is the distance at which the emergent ray cuts the focal plane, measured from the axis; V is the angle which the corresponding ray makes with the axis. A very slight amount (.003 mm.) of residual coma is seen to be present, agreeing with the result already found at F/14. Ray No. 7 is seen to be unsymmetrical as compared with the others. But reference to Fig. 1 shows that this ray cannot traverse the lens unless the stop is in contact with the second negative element. The best position of the stop will be close to the first negative element. In the figure it is shown in a central location. It is very easy to eliminate the small residual coma just found. It does not seem worth while, however, to make a recomputation. For coma is controlled by the ratio of the radii of the first and last surfaces, or R_1/R_8. It

FIG. 1.

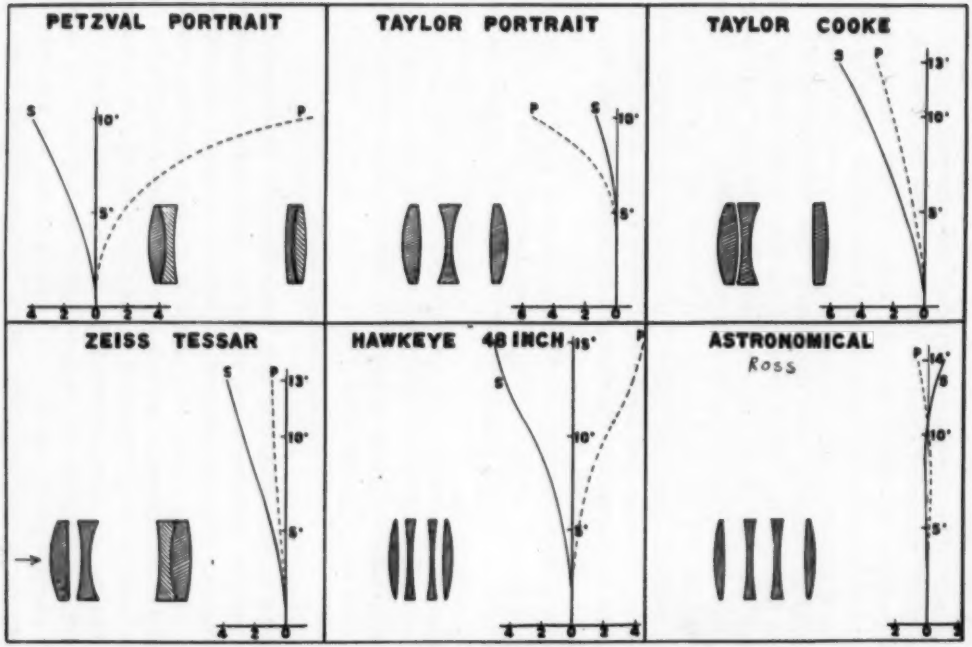


Wide-angle astronomical doublet.

is found by calculation that upon diminishing R_1 by one-tenth of one per cent. and increasing R_8 by the same amount, the effect on coma is −.003 mm., or just sufficient to eliminate the amount now outstanding. But it is doubtful if these surfaces can be ground to this degree of accuracy, so that further elimination of coma would be but a refinement of calculation impossible to realize in practice. If when constructed a lens is found to have coma present, the coma can be progressively "worked out" by small radii changes in R_1 and R_8. If positive coma, for example, is present as in our case (edge zones focusing further from the axis), R_1 should be decreased and R_8 increased equally; if negative coma is present, R_1 is to be increased and R_8 decreased.

Astigmatism.—There is no question but that the amount of residual astigmatism and field curvature in the lens as calculated,

FIG. 2.



Field curves of lenses.

within 12° of the axis at least, is much smaller than can be detected by the most rigid tests. But in manufacturing, astigmatism and accompanying it field curvature are sometimes introduced. They can be very simply and effectively removed by altering the central separation S_2. If, for example, the primary focus is found to be nearer the lens than the secondary focus by one-tenth of one per cent., at a field angle of 10°, it can be corrected by increasing the central separation S_2 by 6 per cent.

Distortion.—Distortion has been corrected by making S_1 and S_3 unequal, or bodily moving the two negative lenses forward by the same amount. Negative or barrel distortion is corrected by moving the negatives to the rear; positive or pin-cushion distortion is corrected by moving them forward. The lens separations and thicknesses can be measured so accurately that there appears to be no danger of introducing any distortion that can be detected by any but the severest tests. In the above lens a forward movement of the negative elements of 0.1 mm. produces a negative distortion of .01 mm. at 10°.

Color.—The lens has been calculated to bring the F and G′ rays into coincidence. Any error due to faulty determination of the dispersion of the glasses or to other causes can be corrected by altering S_1 and S_2 by equal amounts. For example, in the lens as given an undercorrection of 0.1 mm. can be corrected by decreasing S_1 and S_3 by 0.25 mm. There is, however, in this case a considerable change introduced into the focal length, amounting to +4 mm., or 2 per cent. Since, however, the effect of any color error is symmetrical, of the same nature as spherical aberration or unsteadiness in seeing, it is not serious, unless of large amount. The correction should, therefore, not be undertaken.

It will be of interest to compare the field characteristics of the present lens with those of lenses already in use. These are collected in Fig. 2. Only lenses are included which have some claim to be considered wide-angle lenses in the astronomical sense. In the case of the Petzval Portrait and the two Taylor lenses, the data are taken from Von Rohr's "Theorie und Geschichte des photographischen Objective." The Tessar curves are from calculations of the writer. Those of the Hawk-Eye 48-inch are from measured curves. The curves of the "Astronomical," which is the designation of the present lens, are from numerical data given above. It will be noticed that in the case of the Petzval

Portrait and the Hawk-Eye 48-inch, there is considerable astigmatism, but very small field curvature. The Petzval Portrait has proved very satisfactory in astronomical photography over a total field of about 8°. The Taylor Portrait is seen to be nearly perfect over a half-field of 5°, but "breaks" very rapidly from this point on. The Cooke and Tessar are of the same general type, with inward curving field. It is quite likely that the Hawk-Eye 48-inch would give critical definition over a half-angle of 8°. The definition of the "Astronomical" should be critical over a half-angle of 14° or 15°.

ROCHESTER, N. Y.,
November 26, 1920.

THE PRESENT STATUS OF THE CONSTANTS AND VERIFICATION OF THE LAWS OF THERMAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE.*

BY

W. W. COBLENTZ.

CONTENTS.

	PAGE
I. INTRODUCTION	131
II. DISCUSSION OF BLACK BODY RADIATION, RADIATORS, SHUTTERS AND REDUCTION FORMULAS	135
III. DISCUSSION OF VARIOUS DETERMINATIONS OF THE COEFFICIENT OF TOTAL RADIATION	142
IV. DISCUSSION OF VARIOUS DETERMINATIONS OF THE CONSTANT OF SPECTRAL RADIATION	147
V. VERIFICATION OF THE LAWS OF THERMAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE	148
VI. BIBLIOGRAPHY	152

I. INTRODUCTION.

THIS seems to be an appropriate time for a retrospect of past accomplishments and for a new starting point in the determination of the constants of thermal radiation, which investigations, everywhere, have suffered much from interruption during the past six years.

The various determinations of the numerical values of the constants of radiation have been hampered by the lack of exact knowledge of numerous contributory constants which enter into their evaluation.

Among these factors are the temperature scale, the absorptive coefficients and refractive indices of the prisms used, the correction for atmospheric absorption, and the correction for reflection from the radiometer receivers.

When the writer began studying this subject, about fourteen years ago, it appeared desirable to determine, more exactly, the numerical values of the contributory constants, first of all, and then take up the determination of the constants of thermal radiation. At this day it is quite evident that such a procedure would have been the most efficient in obtaining the desired results. While such a program could not be fully carried out, some progress was

* Section of 1920 Report of Standards Committee on Spectroradiometry, W. W. Coblentz, Chairman.

made by determining the diffuse reflection of various substances [1]† (*e.g.*, platinum black), the absorption constants of quartz,[2] and the correction for atmospheric absorption.[3, 4] Somewhat similar experiments have been made by Warburg [5] on the absorptivity of quartz and by Gerlach [6] on the atmospheric absorption.

Instead of a unified program of research, we find that while one set of experimenters was engaged in establishing the temperature scale, another experimenter was determining the optical constants of the prism material and a third group, using both the temperature scale and the optical constants, was attempting to determine the radiation constants. In turn, the first set of experimenters had to apply the results of those working on the radiation constants in order to verify and extrapolate the temperature scale beyond that attainable with the gas thermometer. In the meantime, the second experimenter improved his measurements of the optical constants (refractive indices) of the prism material (fluorite and quartz), while a fourth entered the field and added refinements by determining the temperature coefficients of these refractive indices. This in turn necessitated the recalculation of the prism calibration and the numerical values of the spectral radiation constants. Added to these difficulties is the constant change in personnel, as is apparent from perusal of the title pages of the published papers. The foregoing is an impersonal statement of the difficulties under which experimenters have been laboring. Fortunately, the spectral radiation observations are in terms of the scale of the spectrometer circle, and if the necessity arises, as described on a subsequent page, one can revise the corresponding wave-lengths and recalculate the constants.

As a result of these various difficulties there is scarcely a determination of the constant, σ, of total radiation, and of the constant, C, of spectral radiation, which does not require corrections, some of which are obvious and could have been made by the experimenter.

It seems appropriate to point out specifically some of the difficulties experienced by the two national laboratories.

The Reichsanstalt Investigations.—The determinations of the radiation constants, made at the German Reichsanstalt during the past two decades, involved measurements extending to about 1600° C., or even higher, and hence they have suffered from both

† References will be found at end of text.

the lack of knowledge of the temperature scale and of the refractive indices of the prisms used. In the later work, the determination of the temperature scale was rightfully combined with the measurements of the radiation constants, instead of having the former measurements made in a separate laboratory.

The published papers do not state whether an attempt was made to harmonize the various determinations of the refractive indices of fluorite. The most recent work was done with quartz prisms and the determination of the indices of refraction of quartz was undertaken,[7] but as yet no data have been published.

The Bureau of Standards' Investigations.—The determinations of the radiation constants, especially the more recent ones, made at the Bureau of Standards, have suffered but little from a lack of knowledge of the temperature scale, owing to the fact that the radiators were not operated at a temperature much above 1200° C., and usually near the melting point of copper—1083° C. The chief difficulty has been the lack of exact knowledge of the refractive indices of fluorite; or, as it appears from the present inquiry into the matter, the lack of exact knowledge of the refractive index for the yellow sodium line, and especially the yellow helium line, which was used as a reference point in calculating the infra-red calibration of the prism.

The size of the apparatus used in determining the refractive indices is an important element in the accuracy attained. Langley [8] used a spectrometer, having an image-forming mirror of over 4.5 metres focal length, in an enclosure which could be kept at a constant temperature. Paschen's [9, 10, 11, 12] apparatus had an image-forming mirror of only 1/10 to 1/12 that of Langley's and hence could not compete with it in accuracy. Hence, from the beginning, more weight should have been given to Langley's measurements. On applying Micheli's [13] temperature coefficients of refraction, Paschen's various determinations are found in good agreement with those published by Langley,[8] whose reference point is the solar absorption line (the "A line") $\lambda = 0.7604\mu$; refractive index $n = 1.431020$ for fluorite at 20° C. From this point into the infra-red, the various determinations of the infra-red refractive indices of fluorite are in excellent agreement.

However, the fiducial line in this Bureau's spectroradiometry is the yellow helium line. From the present study of the data it appears that prior to 1913 the refractive indices of fluorite, for

the visible spectrum, and in particular that of the yellow helium line $\lambda = 0.58758\mu$, were not known with sufficient accuracy to meet the present-day demands for refinements in the radiation constants. As a result, erroneous wave-lengths were assigned to the infra-red (minimum deviation) settings of the spectiometer circle. This error is only 3″ (true value, 6″) in the Wadsworth system of spectrometer settings and hence scarcely observable experimentally. However, it enters as a constant error of 0.3 to 0.4 per cent. in the calculations of the calibration curve. Errors of such small magnitude were considered insufficient eight years ago, when there was a disagreement of 1 to 2 per cent., or even more, in the various determinations of the constant of spectral radiation.

As will be shown on a subsequent page, using Paschen's [12] recent measurements of the refractive index of fluorite for the yellow helium line (which measurements were made with a new and very accurately graduated spectrometer), all the apparent errors seem to be eliminated from this Bureau's determination of the constant of spectral radiation which is now in good agreement with the value that is to be expected from this Bureau's determination of the constant of total radiation [15] and from theoretical considerations.[4]

Object of the Present Paper.—In previous papers [15, 4] corrections were made to the determinations of the radiation constants applied by various experimenters. The object of the present paper is to make an up-to-date examination into the various instruments and methods employed in these various determinations, and to give an estimate of the results therewith obtained.

At first glance, the various determinations of these constants appear to cover a wide range of numerical values. This is especially true of the values of the coefficient of total radiation, in which all sorts of ill-considered methods have been used. Moreover, no corrections were made for atmospheric absorption of the thermal radiation in its passage from the radiation to the receiver. As will be shown presently, most of these older determinations are in markedly close agreement with the newer ones after making obvious corrections for atmospheric absorption and for reflection of the incident radiation from the receiver.

This paper does not include a discussion of the attempts which have been made to determine the constant, C, by means of an

optical pyrometer, which at least in its ordinary form cannot be considered an instrument of high precision. There are many difficulties to be overcome before one can conclude that the data so obtained are trustworthy. One of the uncertainties is the effective wave-length of the red glass used in the eyepiece of the pyrometer. Other difficulties are encountered when using a spectral pyrometer. Also the temperature scale is in doubt. Hence recent experimenters have not attempted to determine C, but have reversed the process and, working on the assumption that C is a certain value, say $C = 14350$, have determined the melting point of palladium. This seems to be the preferable procedure.

II. DISCUSSION OF BLACK BODY RADIATION, RADIATORS, SHUTTERS AND REDUCTION FORMULAS.

In order to arrive at an intelligent understanding of what has been accomplished in the determination of the constants of radiation, and in order to indicate the way to future progress in this subject, it is important to consider very briefly some of the conceptions, as well as some of the most recent instruments and methods used in the production and measurement of the radiation from a uniformly heated enclosure or so-called black body.

Every substance has a characteristic emission spectrum. On the other hand, the emission spectrum of a uniformly heated cavity is independent of the composition of the material forming the walls of the enclosure. It is a function only of the temperature.

The emission of thermal radiation from, and the absorption of thermal radiant energy of, all wave-lengths entering into the cavity is complete. In a material substance this can occur only on the short wave-length side of the region of anomalous dispersion where the absorptivity is high but where the reflecting power is practically nil; for example, in quartz,[20] selenite and aluminum oxide, in the region of 7μ to 10μ ($\mu = 0.001$ mm.).

Modern conceptions of black body radiation are the result of slow development from such simple experiments as those of Draper,[19] who found that the interior of a rifle barrel became luminous at a certain temperature; of Christiansen,[21] who observed that the scratches and conical-shaped holes in an incandescent metal surface were brighter than the smooth surface; and of St. John,[22] who found that the selective emission, commonly observed in the rare oxides, disappeared when these substances were heated in a uniformly heated porcelain tube.

Radiators.—The modern uniformly heated cavity or so-called black body is the result of such observations and facts as just enumerated. It is the invention of Wien and Lummer [23] and, for attaining fairly high temperatures, such as are used in many of the investigations mentioned in a subsequent part of this paper, consists of a diaphragmed porcelain tube wound with platinum ribbon, which is electrically heated.

In the modification used by Coblentz [30] the radiator consisted of two concentric tubes suitably wound with platinum ribbon in order to obtain a uniform temperature throughout a great length of the interior of the radiator, as shown by Waidner and Burgess.[31] The use of platinum ribbon instead of thick platinum wire eliminates local non-uniformity of temperature in the porcelain tube.

In radiators of this type the porcelain tubes sag on heating to 1200° C. To overcome this difficulty, in the determination of the radiation constants, to be described presently, Coblentz [30] placed a wedge-shaped fragment of porcelain, with a sharp edge, about 2 mm. long, about 25 to 30 mm. to the rear of the radiating diaphragm. This support causes no local cooling, and it preserves the life of the radiator. Further improvements were made in the completeness of the emission of the radiation by painting the inside walls and the front side of the radiating diaphragm with a mixture of chromium oxide and cobalt oxide. Since these oxides become electrically conducting at 1200°, the front thermocouple was completely inclosed in a porcelain insulating tube. The short piece of porcelain which lies across the radiating wall of the black body was covered with cobalt oxide. Since the cobalt oxide did not adhere well to the porcelain tube, the latter was first covered with a thin layer of iron oxide, obtained by wetting the porcelain tube with writing ink, which was burned into the tube. The cobalt oxide paint was then applied and also burned upon the tube by means of a blast lamp. After replacing the thermocouple, inclosed in this short tube within the radiator, the whole interior was again painted with a mixture of cobalt and chromium oxides.

The cobalt oxide has a high temperature coefficient of absorption so that it appears black on slight heating. Its emission spectrum [29] is continuous, so that there is less difficulty in producing a perfect radiator than when the radiating inclosure is of white porcelain. However, porcelains having a low melting point when heated above 1000° emit a far more continuous spectrum than

the " Marquardt porcelain " from which are made the black bodies ordinarily used.

Water-cooled Shutters and Diaphragms.—In discussing the relative merits of the various determinations of the radiation constants, and in particular the coefficient of total radiation, the writer will have occasion to refer frequently to the use, or misuse, of shields for preventing the receiver from being heated by the radiator, and of shutters for exposing the receiver to radiation. It is, therefore, important to consider briefly the functions of this apparatus.

The determination of the coefficient of total radiation usually consists in noting the heat interchange when the receiver is exposed to two radiators which are at widely differing temperatures, say 0° C. and 100° C., *i.e.*, ice and boiling water. The radiator at the lower temperature may be, and usually is, used as a shutter. If the temperature of the shutter is lower than that of the receiver, the latter radiates to the shutter. Hence, it is important to have the receiver face a large (say, water-cooled) diaphragm, maintained at a constant temperature, back of which is placed the shutter and the radiator. In this manner the surrounding conditions, facing the receiver, are not changed when the shutter is moved in order to expose the receiver to radiation.

There is no doubt that some of the unusual results obtained in the determination of the radiation constants are owing to the operation of the shutter under such conditions that the receiver did not face uniform conditions when the shutter was operated in making the measurements.

Experience shows that the arrangement of the shutter should be similar to that used[3] (*loc. cit.*, p. 516, Fig. 5) in the determination of the coefficient of total radiation. In this arrangement the water-cooled shield employed consisted of a tank, 35 cm. in diameter and 1.5 cm. in thickness, which faced the radiator, and a second tank, 20 cm. in diameter and 3 cm. in thickness, which faced the radiometer. The water-cooled shutter, consisting of a thin metal box 3.5 by 3.5 by 0.8 cm., was operated in vertical ways between these two shields. A mercurial thermometer was used to measure the temperature of the water flowing through this shutter. The side of the shutter facing the radiometer was smoked in a sperm candle, and in connection with the conical-shaped opening (painted black and smoked) in the water-cooled

diaphragm which faced the radiator it formed a miniature black body, the temperature of which remained constant throughout a series of measurements.

Statement of the Laws of Radiation.—The law of total radiation, generally called the Stefan-Boltzmann law,[17] states that the total emission of thermal radiation, R, of all wave-lengths passing from a uniformly heated enclosure, at temperature T_1 to another body at a temperature T_0 is proportional to the difference of their absolute temperatures to the fourth power, or

$$R = \sigma(T_1^4 - T_0^4) \quad \ldots\ldots(1)$$

In this formula, σ is the coefficient or constant of total radiation under discussion. The numerical value of this coefficient is of the order $\sigma = 5.7 \times 10^{-5}$ erg per cm.^{2} per deg.^{4} In this paper only the first three significant figures will be mentioned, *e.g.*, $\sigma = 5.32$.

The law of spectral radiation, which specifies the distribution of thermal emission intensities in the spectrum of the radiation emitted by a uniformly heated enclosure, or so-called black body, is best represented by Planck's [18] formula

$$E_\lambda = c_1 \lambda^{-5} (e^{C/\lambda T} - 1)^{-1} \quad \ldots\ldots(2)$$

In this formula the constant C determines to a great extent the shape of the isothermal spectral energy curve. Suffice it to say that these laws are based on well-grounded theoretical foundations, and after almost two decades of discussion they remain unchanged.

Recently, Nernst and Wulf [78] have arbitrarily modified the coefficient in the Planck equation, changing C_1 to $C_1(1 + \alpha)$ where α is a variable. The whole procedure is based upon the assumption that the numerical value of the constant is $C = 14300$, as observed from isochromatic observations. Hence, in view of the fact that the older values of C obtained from isothermals are somewhat higher, they must be reduced by a factor $(1 - \alpha)$, depending upon the temperature of the radiator. They point out explicitly that the whole procedure depends upon the truth or falsity of the value of $C = 14300$; or rather, as will be seen presently, upon the truth or falsity of the older values of $C = 14400$ (about) obtained from isothermal measurements. Their deductions lead to a value of $\sigma = 6.04$, which, from a consideration of the whole subject, appears to be much higher than the true value. Until we have more reliable experimental data using isothermals, it does not appear necessary to consider a modification of the Planck equation.

The isothermal spectral energy curves published by Coblentz [76] (*loc. cit.*, pp. 474 and 476, Figs. 1 and 2) show that, in the region free from atmospheric absorption to 6μ, the agreement between the observed and the computed curves agree to within the errors of observation, *viz.*, 0.5 to 1 per cent.

Rubens and Kurlbaum [77] have made isochromatic observations, using the residual rays of fluorite in the spectral region of 24μ and 52μ. Their data are also in exact agreement with those computed by the Planck equation. From these data it appears that the Planck equation may be considered as representing within the errors of observation the energy distribution of a black body in the spectrum extending from 0.5μ to 50μ.

Spectral Radiation Formulas.—For computing the constant C from an *isochromatic* energy curve, at any wave-length, Planck's equation is used in the following form: [30]

$$C = \frac{(\log E_2 - \log E_1)\lambda T_1 T_2}{\log e(T_2 - T_1)} - \frac{(e^{-c/\lambda T_2} e^{-c/\lambda T_1})\lambda T_1 T_2}{T_2 - T_1} \quad \ldots\ldots(3)$$

where E_1 and E_2 refer to the emissivities corresponding to the temperature T_1 and T_2, respectively. In this equation the terms $\log (1 - e^{-c/\lambda T_1})$, etc., are expanded in series and only the first term of the expansion $(-e^{-c/\lambda T_1} \log e)$ is used. An approximate value of $C = 14300$ is used in applying the second term correction. For wave-lengths up to 1μ this correction term (to the Wien equation) is small, but beyond 1μ these corrections increase very rapidly with wave-length. This explains the rise in the value of C with wave-length as found by Lummer and Primgsheim,[71] who did not use the second term correction. Energy curves which coincide closely with the Wien equation give uniform values of *C* with increase in wave-length, when computed by using only the first term in equation, as was found by Paschen.

Using an *isothermal* spectral energy curve, the constant of spectral radiation C (eq. 2) may be derived from the value of the wave-length of maximum emission, λ_m, by means of the equation:

$$C = \alpha \lambda_m T \quad \ldots\ldots(4)$$

in which the value of $\alpha = 4.9651$. This equation makes use of the Wien displacement law, which is the mathematical expression for the shifting of the wave-length of the maximum emission toward the shorter wave-lengths, with rise in temperature, *viz.*,

$$\lambda_m T = A \text{ (a constant)}. \quad \ldots\ldots(5)$$

The value of the constant is close to A = 2885 micron degrees.

In case one cannot observe the complete energy curves, as, for example, when using a quartz prism which absorbs heavily beyond 2.2μ, it is possible to compute the value of λ_m from the Planck equation by reducing the observed energy curve to the normal spectrum and taking from the energy curve the values of E_1 at λ_1 and E_m. The proper equation is:

$$\frac{E}{E_m} = \left(\frac{\lambda_m}{\lambda}\right)^5 \frac{(e^{4.9651} - 1).}{(e^{4.9651\lambda m/\lambda} - 1)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

This equation, which has been extensively used by Warburg [86] and his collaborators, appears to be well adapted when using a quartz prism; but the temperatures must be sufficiently high so that the E_m does not fall in the region beyond 2.2μ, where quartz begins to absorb heavily. This equation is not well adapted to a fluorite prism owing to the small dispersion of the prism, and hence the steepness of the energy curve on the short wave-length side where one would want to use it, and also owing to the fact that, at the temperatures usually employed, the E_m falls in the region of atmospheric absorption bands or in the region of 1.6μ where the dispersion curve of fluorite has a point of inflection. The latter makes it quite difficult to determine accurately the factors for reducing the observations from the prismatic to the normal spectrum.

Another method of determining the values of λ_m from an isothermal spectral energy curve is by the "method of equal ordinates." For this purpose the normal spectral energy curve is drawn on accurately ruled coördinate paper and at two points where the intensities (the galvanometer deflections) are equal, $E_1 = E_2$, the corresponding wave-lengths λ_1 and λ_2 are read from the curve. The value is easily derived [80] from equation (2) by equating $E_{\lambda_1} = E_{\lambda_2}$ and for the complete solution it is:

$$\lambda_m = \frac{5(\log \lambda_2 - \log \lambda_1)\lambda_1\lambda_2}{\alpha(\lambda_2 - \lambda_1)\log e} - \frac{\lambda_1\lambda_2[\log(1 - e^{-c/\lambda_2 T}) - \log(1 - e^{-c/\lambda_1 T})]}{\alpha(\lambda_2 - \lambda_1)\log e} \qquad (7)$$

The second term in this equation can usually be abbreviated, since terms involving λ_1 are usually negligible. For values of λ_2, which are less than 4μ, the term $\log(1 - e^{-c/\lambda_2 T})$ may be expanded into a series and (by dropping all terms but the first) may be used in the form $-e^{-c/\lambda_2 T}\log e$. In this equation $\alpha = 4.9651$. It may be noticed that an approximate value of C is necessary in

order to obtain a solution for the second-term factors in equation (7). This may be obtained by solving for λ_m by using only the first term in equation (7), which is the solution of the Wien equation as used by Paschen.[69] It is sufficiently accurate [30] to use $C = 14300$ in making this computation. A decrease of 100 units in C (say, $C = 14200$) decreases the mean value of λ_m by 0.0012μ.

The method of equal ordinates was extensively used by Paschen [69] and by Coblentz.[30, 76] It has several commendable features because it is possible to utilize values of λ_1 and λ_2, which correspond with values of E_{λ_1} and E_{λ_2}, which are closely the same in magnitude as originally observed, and hence contain the same errors of observation; also the slit widths are closely the same. This method does not require the most accurate wave-lengths and refractive indices (at least not for values of λ_2 greater than 4.5μ) and it permits the selection of parts of the spectral energy curves which are free from atmospheric absorption bands.

The method of equal ordinates necessitates reducing the prismatic spectral energy measurements to a normal spectrum, plotting the data upon coördinate paper and drawing in a smooth curve from which the values of λ_1 and λ_2 are read, corresponding with the equal ordinates E_{λ_1} and E_{λ_2}. Although it requires but little additional time to plot the data after having made the observations, if one is certain that the observations lie close to the curve, the obviously logical procedure is to compute the spectral radiations constants from any two observed points E_{λ_1} and E_{λ_2}. This, however, does not shorten the observational work, for, as in previous investigations, it will be necessary to make observations at three adjacent spectrometer settings (separated by the width of the radiometer receiver) in order to determine the spectral purity factor which is required in order to reduce the data to a normal spectrum. After spending months in obtaining the data, the question of saving time in their reduction is of minor importance. It is desirable to draw the complete curve and preserve it for future reference. If the observations do not lie close to the smooth curve, then this method is slightly arbitrary as to the manner of combining the observations so as to obtain an integration of the whole curve without making more computations than would be used in the "method of equal ordinates."

The appropriate formula for computing the radiation constants from any two observed points E_{λ_1} and E_{λ_2} on an isothermal

spectral energy curve is easily deduced [76] from Planck's equation, and in its complete form, as shown by Dellinger,[80] is:

$$C = \frac{\lambda_1\lambda_2 T}{(\lambda_2 - \lambda_1)\log e}\left[\log\frac{E_{\lambda_2}}{E_{\lambda_1}} + 5\log\frac{\lambda_2}{\lambda_1} - \log\left(\frac{1 - e^{-c/\lambda_1 T}}{1 - e^{-c/\lambda_2 T}}\right)\right] \quad \ldots (8)$$

The term $\log (1 - e^{-c/\lambda T})$ may be expanded into a series and usually all the factors can be neglected, except one which is $\log e \cdot e^{-c/\lambda T}$. It is therefore necessary to know the approximate value of C, as was required in Coblentz's [30, 76] method of computation.

In concluding this discussion we may notice a calculation of the constant C by an extension of the theory of least squares by Roeser.[81] His calculation gave a value of $C = 14342$, as compared with $C = 14339$ computed by Coblentz [76] (*loc. cit.*, p. 462), by the method of equal ordinates.

A spectrophotometric method may also be used in determining the constant C. It consists in determining the ratio of brightness of a black body at, say, the melting point of gold (1063° C.) and at some higher temperature, say the melting point of palladium (about 1556° C.). This requires a knowledge of the wave-length ($\lambda = 0.65\mu$ is usually taken) of the light photometered and the absolute temperatures T_1 and T_2 of the radiator. The appropriate formula is:

$$\log\frac{E_2}{E_1} = C\frac{\log e}{\lambda}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \quad \ldots (9)$$

The intensities E_1 and E_2 are usually measured with an optical pyrometer. If the constant C is known the temperature T_2, say 2000° C., may be computed. Conversely using known temperatures T_1 and T_2 the constant may be determined. As a matter of fact, the method has never been used very successfully.

III. DISCUSSION OF VARIOUS DETERMINATIONS OF THE COEFFICIENT OF TOTAL RADIATION.

It has been shown by Stefan,[16] Schneebeli,[25] Wilson,[26] Lummer and Pringsheim,[24] Valentiner,[27] and others,[3] that the total radiation emitted from a uniformly heated cavity is proportional to the fourth power of the absolute temperature. The measurements extend over a temperature range of perhaps 1500° C. They verify the fourth power law within the accuracy of the temperature scale, and the general errors of observation which are small owing to the fact that the measurements are relative.

The evaluation of the coefficient of thermal radiation in absolute measure is an extremely difficult task. It has been attempted by numerous experimenters, whose data are fully considered in a paper by Coblentz,[10] an abstract of which is given in the following pages, and summarized in Table I.

The early determinations of the coefficient of emission of a substance (*e.g.*, blackened plates or balls of glass or copper) by Lehnebach,[34] Christiansen[35] and others are of great historical interest, but they cannot be considered in connection with present-day determinations of the coefficient, or constant σ, of total radiation of a uniformly heated enclosure.

1. *Bolometric Methods of Electric Compensation.*

The principle of the method is as follows: Three branches of a Wheatstone bridge (bolometer) consist of thick manganin wires, which are not affected by a change in current through the bridge. The fourth branch consists of thin bolometer platinum, the resistance of which is affected by a change in current in the bridge. Starting with the bridge balanced, the bolometer branch of thin platinum is exposed to the radiation from a black body and the change in resistance (or galvanometer deflection) is noted. With the bolometer branch shielded from radiation, the bridge current is varied until the change in resistance in the bolometer branch is equal to that attained when exposed to radiation. In other words, the bridge current is increased by a sufficient amount to produce the same galvanometer deflection as was caused by radiation falling on the bolometer strip. From a knowledge of the change in the bridge current, the bolometer resistance, etc., the observer is able to compute the energy input.

Observations by this method have been made by Kurlbaum[36] and by Valentiner.[27] The value of $\sigma = 5.32$, first obtained by Kurlbaum, has been corrected[37] for reflection, giving a value of $\sigma = 5.45$. Valentiner[38] corrected his measurements for reflection from the receiver and for lack of blackness of the radiator, raising his original value from $\sigma = 5.36$ to $\sigma = 5.58$. Coblentz[107] adds corrections for atmospheric absorption, thus increasing the value to $\sigma = 5.68$, with a possibility of the value being $\sigma = 5.75$.

Observations have been made also by Gerlach,[42, 48, 106] by Coblentz,[3, 15, 4] and by Kahanowicz,[54] using modified forms of the Ångström pyrheliometer.[43]

In the instrument used by Gerlach and by Coblentz there is but one strip of manganin which is about 0.01 mm. in thickness, and 2 to 8 mm. wide (length about 3 cm.). At the back of this manganin strip and close to it is a thermopile (joined through a galvanometer), which is heated by radiation from the manganin strip, thus differing from Ångström's device in which a single thermojunction was heated by conduction from the manganin strip. Coblentz attached the potential terminals directly to the receiver at a sufficient distance from the ends to avoid the question of heat conduction to the electrodes. These potential wires, which are from 0.003 to 0.025 mm. in diameter, accurately define the length of the central part of the receiver which was utilized in the measurements. By exposing the whole length of the receiver to radiation conduction losses from the end of the receiver do not enter the problem.

Gerlach's original value was $\sigma = 5.9$. Subsequently he repeated the work and obtained a value of $\sigma = 5.85$, which is now revised [106] to $\sigma = 5.80$. Coblentz [4] has recalculated his data, the original value [15] remaining unchanged at $\sigma = 5.72$ (see Table I).

Kahanowicz [54] used a receiver at the centre of a spherical mirror and obtained a value of $\sigma = 5.61$. Coblentz has corrected [4] this value for atmospheric absorption and obtained a value of $\sigma = 5.69$ to 5.73.

Puccianti [53] determined the constant, σ, by means of a very ingeniously constructed black body bolometer, one branch of which was exposed to a receiver which was cooled by carbon dioxide snow or liquid air and the other was protected from it. These bolometer branches consisted of vessels of thin sheet copper having the form of a cone and a frustrum of a cone united at the bases. The internal surface was smoked. The external surface was polished and upon it was wound two thin insulated wires. One of these wires, of iron, formed the bolometric branch, and the other, of manganin, was used as a heating resistance. The other two branches of the bolometer bridge were formed of resistance coils and the whole was connected with a galvanometer and storage battery as in any ordinary bolometer.

Puccianti measured the (electrical) energy necessary for compensation to prevent the bridge from being unbalanced when one branch was exposed to the receiver. He obtained a value of $\sigma = .596$.

2. Thermometric Methods with Black Receivers.

Numerous determinations of the coefficient of total radiations have been made by means of conical or spherical-shaped receivers formed into thermometer bulbs. The space surrounding the receiver was filled with a liquid, *e.g.*, aniline, which served as a thermometric substance whose expansion could be observed in a capillary tube. The method of operation consists in exposing the receiver to radiation and noting the thermometer reading. Then a coil of wire is heated electrically to give the same thermometer reading.

The unsatisfactory part of the arrangement is the heating coil. In some cases the heating coil was wound on the outside of the conical receiver, and, as shown in the critique by Coblentz,[107] an erroneous (too high) value of the radiation constant is obtained. This is the chief criticism against the measurements of Féry [55] ($\sigma = 6.30$), of Féry and Drecq [46] ($\sigma = 6.51$) and Puccianti [61] ($\sigma = 6.15$). When the thermometer is calibrated by placing the heating coil within the (spherical-shaped) receiver, as used by Keene [62] ($\sigma = 5.89$) the arrangement appears to be free from criticism.

Keene's receiver and the radiator being close together, and the time for attaining temperature equilibrium being rather long, there is a possibility of diffusion of hot gases into the receiver when the shutter is raised for observations. This would tend to give a high value. It would have been interesting to determine whether it would have required less electric power in the heating coil, provided the opening in the receiver had been closed with a reflecting cover to prevent escape of heated gases, while calibrating the device.

3. Indirect and Substitution Methods.

Shakespear [63] determined the constant of total radiation from a series of experiments in which a plate of metal with a silvered surface was heated electrically to 100° C. and close to it was another plate, blackened with soot, which was cooled with water. Between the plates was air at atmospheric pressure. Shakespear measured the energy input in order to maintain the plate at 100° when (1) the surface of the plate was highly polished, and (2) when it was blackened. He measured also the ratio of the emissivities of the plate under these two conditions, using a radiomicrometer for the purpose. He obtained a value of $\sigma = 5.67$.

Todd [64] obtained indirectly a value ($\sigma = 5.48$) of the coefficient

of total radiation from his experiments on the thermal conductivity of gases, in which he used a thin layer of air, enclosed between two horizontal, parallel, good conducting plates, which were maintained at different temperatures.

It is interesting to find that these data, which were obtained by widely different methods, and which appear so discordant on first perusal of the original papers, are in excellent agreement. The various values range about the number $\sigma = 5.7$. In fact, two-thirds of the total number (12) of determinations are close to $\sigma = 5.7$. The mean value of all the data which are free from obvious errors is $\sigma = 5.72$ to 5.73. If we neglected Todd's low value, determined from gas conduction experiments, and Puccianti's high value, which is no doubt too high, the mean value remains unchanged. As we shall see presently, experimental evidence on ionization potential, X-rays, and photoelectric work shows that the value of the coefficient of total radiation is of the order $\sigma = 5.7 \times 10^{-5}$ erg cm.$^{-2}$ deg.$^{-4}$ sec.$^{-1}$.

TABLE I.

Observed Value and the Most Probable Value of the Constant of Total Radiation After Correcting for Reflection, Atmospheric Absorption, etc.

Observer.	Date.	$\sigma \times 10^5$	Probable value of $\times 10^5$	Method.
Kurlbaum	1898	5.45	?	Bolometer.
Féry	1909	6.3	?	Thermometer.
Bauer and Moulin	1909	5.30	5.7	Thermopile.
	1910	5.7	5.7	Pyrheliometer.
Todd	1909	5.48	5.48	Gas conduction expts.
Valentiner	1910	5.58	5.68 to 5.75	Bolometer.
Féry and Drecq	1911	6.51	?	Thermometer.
Féry and Drecq	1912	6.2	5.68	Calibrated pyrometer.
Féry and Drecq	1912	5.57		
Shakespear	1912	5.67	5.67	Ratio of emissivities, metal; black body.
Gerlach	1916	5.85	....	Modified Ångström.
	1920	5.80	5.80	Pyrheliometer.
Puccianti	1912	5.96	5.96	Bolometer.
		6.15	?	Thermometer.
Westphal	1916	5.67	5.67	Ratio of emissivities, metal; black body.
Keene	1913	5.89	5.89	Thermometer.
Coblentz	1915	5.72	5.73	Modified Ångström pyrheliometer.
Kahanowicz	1919	5.61	5.69 to 5.73	Modified Ångström pyrheliometer.
Foote	1918	5.70	5.70	Marvin pyrheliometer.

Mean value, $\sigma = 5.72$ to 5.73×10^{-5} erg cm^{-2} deg^{-4} sec^{-1}

IV. DISCUSSION OF VARIOUS DETERMINATIONS OF THE CONSTANT OF SPECTRAL RADIATION.

Planck's [18] equation is, no doubt, the closest representation yet formulated of the observed energy distribution in the spectrum of a black body. The constant C (often written in the more cumbersome form C_2) determines, to a great extent, the shape of the isothermal spectral energy curve.

The formula is based on theoretical considerations, and after almost two decades of discussion it remains unchanged.

From the experimental data of Coblentz,[76] and of Rubens and Kurlbaum,[77] it appears that the Planck equation may be considered as representing, within the errors of observation, the energy distribution of a black body in the spectrum extending from 0.5μ to 50μ.

In spite of all that has been published on the partition of energy in the spectrum of a black body, there are but few data at hand which are more than qualitative in value.

It seems that all these early data should be considered from the standpoint of historical interest, since it seems impossible to give them much weight in connection with the results obtainable at the present day.

The pioneering work in spectral radiation measurements and constants was inaugurated by Paschen,[69, 70] who observed an extensive series of spectral energy curves.

In a recent critique [107] it is shown that Paschen's original data fall within the range of the recent and more accurately determined values, being of the order $C = 14360$.

In this same critique,[107] it is shown that the observations of Lummer and Pringsheim,[71, 73, 83] after corrections are applied to their calibration of the prism, yield a value of $C = 14300$.

Observations of Warburg and collaborators: [86-95] During the past twenty years the determination of the constants of radiation and their application to the temperature scale has been relentlessly pursued by the *Phys. Tech. Reichsanstalt*,[83] at Berlin.

During the past ten years these investigations have been pursued by Warburg and his collaborators. They used fluorite and quartz prisms, and vacuum radiators. The sodium line was used as a zero setting of the spectrometer. The radiometer was a vacuum bolometer, which was operated by a null method. In this manner the galvanometer acted merely as an indicator. The

temperature fixed points were the melting point of gold and some higher temperature, *e.g.*, melting point of palladium. This has caused some difficulty, as has also the question of the dispersion curves of quartz. Their data are summarized in Table II. The present status is the adoption[95] of the value of C = 14300 and the melting point of palladium = 1557° C. Subsequent investigations[96, 97] appear to be made on this basis.

Observations of Coblentz:[30, 76, 4, 107] From the very beginning of this investigation on black-body radiation, it was found that the Wien equation did not fit the spectral energy curves. The calculations were therefore made on the assumption that the observed curves fit the Planck equation; and, at the completion of the investigation, this was found to be true for about 75 per cent. of all the observed curves. This conclusion was based upon the uniformity of the values of λ_m, which resulted from computation (by the method of equal ordinates) of values of λ_1 and λ_2, which were taken far apart, and also close together on the observed isothermal spectral energy curve.

The data are summarized in Table II. The main difficulty appears to have been the lack of exact knowledge and coördination of the indices of refraction of the fluorite prism used in obtaining the spectral energy curves. In the recent survey[107] of the whole subject it is shown that after making a small correction to the calibration curve, the value is $C = 14318$.

The various determinations of the constant of spectral radiation, C, are assembled in Table II. The sixth column gives the probable value, as determined from consideration of the data in the text. The mean value is C = 14320 micron degrees. The latest and most reliable determinations of the national laboratories are close to this value. Unfortunately perhaps this average value is too close to the theoretical value, arrived at from consideration of photoelectrical and similarly related phenomena, and experimenters may consider their task finished instead of just begun.

V. VERIFICATION OF THE LAWS OF RADIATION.

In the foregoing pages an inquiry was made into the instruments and methods used in, and the numerical values of, various determinations of the constants of radiation.

The various methods are classified, and a brief description is given of each research.[107] An attempt is given to indicate the good

and the defective features in each research. This represents not only the writer's opinions but also those of other experimenters.

It is shown that the great variation in the various determinations of the numerical values of the constants, especially the constant of total radiation, is owing to the fact that in the original papers no corrections were made for atmospheric absorption of radiation in its passage from the radiator to the receiver.

In this paper conservative corrections for atmospheric absorption are made to the various determinations in which such corrections had not been made. As a result, instead of wide variations in the various determinations, which are free from other obvious defects, there is a remarkably close agreement among the various

TABLE II.

Observed Value and the Probable Value of the Constant C of Spectral Radiation.

Observer.	Date.	$\lambda_m T = A$ observed.	$\lambda_m T = A$ corrected.	$C = a\lambda_m T$ and from isochromatics.	C probable value.	Remarks.
Paschen.....	1899	2891	2891–	.....	14360	Fluorite prism.
		2907?	2907?	.....	.....	Temperature scale is questioned.
	1900	2921?	2894			
Lummer and Pringsheim...	1900	2879	2879	14290	.....	Fluorite prism; wave-length calibration of prism is questioned.
		2876	2876	.....	.....	
		2940	2882	14310	14300	Also temperature scale.
Warburg and Collaborators	1911	....	.....	14200 to 14600	.....	Fluorite prism.
	1912	....	.....	14300 to 14400	.....	Fluorite prism.
	1912	....	.....	14360	.....	Quartz prism.
	1913	2894	.....	14370	.....	Quartz prism.
	1915	....	.....	14250	.....	Temperature from Stefan-Boltzmann law.
	1915	....	.....	14300 to 14400	14300	Refractive indices; calibration of prism questioned
Coblentz.....	1913	2911	....	14456	.....	Fluorite prism.
	1916	2894	....	14369	.....	Revised calibration (refractive indices) of prism; 1912 data are recalculated.
	1920	....	....	14311 to 14326	14318	Correction for zero setting of bolometer.

Average value, C = 14320 micron degrees.

numerical values. While it cannot be said that the true numerical values are exactly as here recorded, obviously the time is past when the value of the constants of radiation are swayed by a single and perhaps novel method of research.

The best that an experimenter should expect is that his own little contribution to the subject may have sufficient merit to go into the melting pot with the other determinations.

The Formula and the Coefficient of Total Radiation.—In the foregoing review the data are assembled, and the evidence weighed pro and con. It is shown from various experiments that, beyond all reasonable doubt, the total radiation emitted from a uniformly heated enclosure is proportional to the fourth power of the absolute temperature—the so-called Stefan-Boltzmann law. Furthermore, the tabulated data show that the numerial values of the majority of the various determinations of the coefficient, σ, which enters into the mathematical formula of total radiation, range about the value, $\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} deg^{-4} sec^{-1}. The average of 12 of the most reliable determinations is $\sigma = 5.72$ to 5.73×10^{-5} erg cm^{-2} sec^{-1} deg^{-4}.

The Formula and the Constant of Spectral Radiation.—Experimental evidence is cited showing that throughout the spectrum from 0.5μ to 50μ Planck's formula fits the observed spectral energy distribution more closely than any other equation yet proposed. This formula is based upon theoretical principles and after two decades of discussion it remains unchanged.

The constant, C, which determines the slope of the spectral energy curve, has been the subject of numerous investigations. The numerical value of C has fluctuated greatly in the various determinations. In the foregoing pages it is shown that this is owing to experimental difficulties, such as, for example, lack of precise knowledge of the temperature scale, and of the refractive indices of the prisms used. The tabulated data show that the various determinations of the constant of spectral radiation are of the order of $C = 14300$ to 14350, with a mean value of $C = 14320$ micron degrees.

Confirmatory Evidence.—One of the most interesting phases of the inquiry into the laws and constants of radiation is the confirmatory data which one obtains from a consideration of the interrelated phenomena of atomic structures of X-rays, of ionization and resonance potential and of photoelectrical action. From

these data, as well as from the foregoing data on the two constants of radiation, one can compute the value of Planck's element of action, h. This gives seven independent methods of determining the universal constant, h. Or, from any one of four of these methods one can calculate [103, 104, 105] the radiation constants, and it seems truly remarkable how close the agreement is with the observed values.

For example, Coblentz's [4] value of the coefficient of total radiation, discussed on a preceding page, is $\sigma = (5.72 \pm 0.012) \times 10^{-5}$ erg cm^{-2} sec^{-1} deg^{-4}. This indicates [105] a value of $C = 14320$ micron degrees and a value of $h = 6.55 \times 10^{-27}$ erg sec. The value of h, determined by Blake and Duane,[102] by X-rays, is $h = 6.555 \times 10^{-27}$ erg sec, or an indicated value of $C = 14330$ micron degrees, which is close to experimental determinations of this constant (see Table II).

Again, starting with Warburg's value of $C = 14300$, the corresponding value of the coefficient of total radiation is $\sigma = 5.74$, which is the higher estimated limit of the average of 12 different determinations of this constant. Hennig,[103] on the basis of Sommerfeld's theory, the measurement of spectral lines and the value of the electron, obtains $C = 14320$ and $\sigma = 5.717$.

In summing up the evidence, it is of interest to include Birge's [104] comprehensive and exact calculations of the constant h. In these calculations he, of course, assumes the truth of (1) Lewis' and Adam's [67] theory of ultimate rational units; (2) of the relation between C, σ, and h, as given by Planck's radiation formula; (3) of the quantum relation as applied to X-ray data;[102] (4) of Einstein's photoelectric equation; (5) of Bohr's theory of atomic structure; and (6) of the quantum relation as applied to ionization and resonance potentials.

TABLE III.

Birge's [104] Calculation of Planck's Universal Constant h by Various Methods.

Value of $h \times 10^{27}$	Method.	Remarks.
6.551 ± 0.009	$\sigma = 5.72$	Total radiation.
6.557 ± 0.013	$C = 14330$	Spectral radiation.
6.542 ± 0.011	Rydberg constant.	Bohr's theory of atomic structure.
6.578 ± 0.026	Photoelectric equations.	Einstein's equations.
6.555 ± 0.009	X-rays.	Quantum relation.
6.560 ± 0.014	Ultimate rational units.	Theory of Lewis and Adams.
6.579 ± 0.021	Ionization potential.	Quantum relation.

Mean value, $h = (6.5543 \pm 0.0025)\ 10^{-27}$ erg sec.

In this manner he obtains seven separate calculations of Planck's constant of action, h, the mean value of which, as shown in Table III, is $h = 6.5543 \times 10^{-27}$ erg sec.

This is close to the average of the value of h, which results from consideration of the two radiation constants.

From this calculation and intercomparison by Birge [104] of the data on C, σ, and h, as determined by thermal radiometric, photoelectric, X-rays, and ionization potential measurements, it appears that the value of h, computed from radiometric data, compares very favorably with that obtained by more direct measurement. In other words, it appears to prove the validity of laws of radiation and to establish the level of the numerical values of the constants entering therein.

The outstanding disagreement between all the observed and computed data appears to be of the order of 1 to 3 parts in 1000, whatever the method of experimentation. This is a very close agreement, considering the variety of the data and the difficulties involved in making the experiments, which seems to indicate something more than a fortuitous relation between properties of matter.

In conclusion it may be added that, to a close degree of approximation, we have the following constants:

$A = \lambda_m T = 2885$ micron degrees
$C = 14320$ micron degrees
$\sigma = 5.72 \times 10^{-5}$ erg cm^{-2} sec^{-1} deg^{-4}
$h = 6.55 \times 10^{-27}$ erg sec
Melting pt. of Pd $= 1556°$ C.

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SPECTRORADIOMETRIC STUDIES OF POLARIZATION PHENOMENA.*

BY

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1. *Magnetic Rotation* [1-7] *Method of Measurement* [6].—Light from a Nernst glower, or a strip filament tungsten lamp, is polarized by passage through a double-image prism (one beam only being used) and converged by a concave mirror to a focus between the poles of the electromagnet. After reflection from another concave mirror it passes a large Wollaston double-image prism whose principal planes are at 45° to the initial plane of polarization. Accordingly, two images of the source of equal intensity but polarized at right angles are formed, and these fall vertically above one another on the slit of the spectrobolometric part of the apparatus. This comprises a mirror spectrometer with large glass prism (45° angle), Wadsworth mirror and special bolometer, whose two strips (½ mm. wide) lie vertically above one another and receive the two spectra resulting from the two images on the slit. The galvanometer is of the Thomson type of 4 or 8 coils with a working sensibility of about 3×10^{-10} amp./mm. for 6 ohms resistance.

The material under examination is placed in the path of the beam between the poles of the magnet. When the latter is excited the rotation of the plane of polarization—which in practice is always small—causes a diminution in the intensity of the beam incident on the upper half, say, of the slit and a corresponding increase in the other. The result is a galvanometer deflection which is doubled by reversing the magnet current. To interpret this in degrees, the polarizing prism is given an accurately known rotation of one or two degrees and a comparison of the galvanometer deflection which this produces with that due to the magnetic rotation allows the calculation of the latter.

The possibilities of this method in measuring very minute magnetic rotations considerably exceed those of visual methods. A rotation of only .001° is readily detected, and, if desired, the sensibility

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can be pushed much farther than this. The accuracy of measurement for rotations of a few minutes of arc is of the order of 1 per cent. In spectral range the limits are about .5μ to 2.3μ, the lower limit being set by energy considerations and the upper by the absorption of the calcite and glass. A simplification of this method, following somewhat more closely the original scheme of Provostaye and Desains and using as polarizer and analyzer reflecting surfaces of glass—or better, as suggested by Pfund, selenium—would, at some expense in sensibility, enable one to proceed farther into the infra-red. The writer's first experiments were carried out along this line, using a rock salt prism and, as polarizer, piles of cover glass plates.

Results: (A) *Faraday Rotation.*—The magnetic rotatory dispersion of CS_2 has been measured [4] to 4.3μ and found to be well represented by a formula based on the Hall effect and taking account of an absorption band beyond 8μ. Water,[5] which may be tested about as far as 1.3μ, shows a similar curve, but one which does not fit the formula as well.

Films of iron, cathodically deposited, show,[6] on the other hand, an increase of rotation for longer wave-length, followed by a decrease. The curves are, in general, somewhat similar to those obtained on reflection (Kerr effect) but of opposite sign.

(B) *Kerr Effect.*—Iron, nickel, cobalt and magnetite show [4] on reflection a maximum of rotation in the neighborhood of 1μ and a decrease for longer wave-lengths. Nickel reverses its sign of rotation at about 1.5μ. These results support the view that the visible spectrum corresponds to a region of anomalous dispersion for such metals. This point of view is somewhat strengthened by other evidence. Heusler's magnetic alloy shows no Kerr effect.

The above measurements on the Kerr effect are for normal magnetization and normal incidence. When the magnetization is transverse and the incidence is varied,[8] the effect is more complicated, but may be shown to be in good agreement with Wind's [9] and also with Voigt's [10, 11] magneto-optic theory.

2. *Natural Rotatory Dispersion* [12-15].—A slight modification of the foregoing method enables the measurement of natural rotation of the plane of polarization over this same spectral region, *viz.*, from .5μ to 2.3μ, with an accuracy only slightly less than that obtained in the case of magnetic rotation. Quartz, widely

investigated by other experimenters,[12, 13, 14] shows a rotation dispersion over this spectrum range which is extremely well represented by a dispersion formula involving two free periods, one in the ultra-violet and the other in the infra-red. Two terpines and three solutions—tartaric acid, cane sugar and camphor—have also been tested in this way as far as their absorption permits. The results, as a whole, are in substantial agreement with the Wiedemann Law of the proportionality of magnetic and natural rotations, the best agreement being in the case of quartz and the least satisfactory for the solution of camphor in alcohol.

3. *Optical Constants of Metals*[16, 17].—The same spectrobolometric apparatus serves here as in the other work. The other arrangements, however, must be considerably modified. The polarized radiation is allowed to fall on the metal plate under investigation in various azimuths and at various angles of incidence. When both azimuth and incidence are so adjusted that the radiation coming from the plate is circularly polarized, the angles of incidence and azimuth are the so-called principal angles and the refractive and absorption indices can be readily computed. To enable one to determine when the radiation is circularly polarized, the metal surface, polarizer, source of light, etc., are all mounted in a massive framework, which can be rotated about the axis of the emergent beam. If the polarization, instead of being exactly circular, is elliptical, this rotation of the ellipse will produce a varying galvanometer deflection, which reduces to zero upon proper adjustment of the incidence and azimuth angles.

Results.—Steel, cobalt and nickel, all show[16] a considerable increase of the refractive index with longer wave-lengths and only relatively small changes in the absorption index. Silver and copper, whose indices of refraction are less than unity for visible wave-lengths, both show a decided increase in the infra-red, copper reaching the value unity at 2.25μ and silver approaching it. Silicon shows a slight decrease in refractive index with longer wave-lengths and a relative constancy of absorption index.

4. *Polarization by Gratings*[18, 19].—When light from a speculum grating is passed through the double image prism and the radiation examined for each state of polarization, certain phenomena are observed. For the radiation whose electric vector is perpendicular to the rulings there is found a sharp minimum of energy for such wave-lengths as are, in some order of spectrum,

passing tangentially from the grating. Such minima are observed either in the central image[19] or in spectra (work not yet published) of the first, second, etc., order, and are, as a rule, more easily studied in the early infra-red (λ about 1μ) than in the visible spectrum. At normal incidence the minimum in the directly reflected radiation for a wave-length equal to the grating space (first order spectra tangential) is very marked.[19]

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SYSTEMS OF COLOR STANDARDS.

BY

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THE principal object of a color standard is to have some means by which the color of any object can be determined. This can be done either by an instrument such as a colorimeter or a standard composed of colored cards. As any instrument is too complicated for general use, only those standards composed of colored cards will be considered in this paper.

The purpose of a color standard and requisites to which it must conform will be first considered, after which the good points and defects of various standards will be taken up.

As was stated, the principal object of a color standard is to have some means by which the color of any object we may happen across can be determined. To do this it is necessary that the cards in the standard be properly arranged so that we can find as easily as possible the card similar to the color in question. As in most instances we wish to record what that particular color is we should have the simplest and best nomenclature. There should moreover be a sufficient number of cards, so that we can find any color. As such a number of cards will tend to make the standard bulky, it should contain the largest possible range of colors with the fewest cards. And last, though very important, every color in the standard should be based on exact measurements, so that people at other times and places will know exactly the color of any card we may designate.

This makes five requisites which a color standard of this kind must satisfy. First, *Arrangement*. It must be such that it is as easy as possible to find any desired color in the standard; that is, such that you know a color's particular position in the standard and its direction from any other color. Second, *Notation or Nomenclature*. The notation of the different cards must be as simple as possible. It should call to mind a particular color and only that color, and should designate its particular position in the standard. Third, *Number*. There should be a sufficient number of cards, so that the color of any object can be closely matched. Fourth, *Spacing*. The spacing between the cards in all parts of

the standard should be the same. With even spacing between all the cards a standard with a given number of cards will have the greatest usefulness. Fifth, *Standardization*. The exact nature of the color in every card should be determined by accurate measurement, so that when a particular card is named its particular color and no other will be designated. Furthermore, such notation and measurements should be universally accepted, so that a particular color will mean the same thing to all peoples at all times.

These five requisites will now be considered in more detail.

ARRANGEMENT.

All colors lie in a natural arrangement to each other, due to the three constants which every color possesses. These are, first, Hue (or wave-length); second, Chroma (saturation); third, Value (brightness).

We are indebted to Munsell for the most complete presentation of their mutual arrangement due to these three constants. He shows their arrangement is in the form of a color solid, roughly spherical in shape. Steps around the horizontal equator give differences in hue or wave-length. Planes at different heights along the vertical axis give different steps in value or brightness, white at the top, black at the bottom. Radial distances from this vertical axis give differences in chroma or saturation.

For practical reasons the cards in a color standard must be arranged on a sheet, *i.e.*, on a two-dimensional surface. It therefore becomes necessary to decide what arrangement there exists in a two-dimensional surface of this color solid which is most suitable for our purpose. It would seem without question to be best to use that grouping which would leave one of the color constants unchanged and the other two variable. Three such groupings, however, can be obtained in a two-dimensional surface.

You can get a grouping in which the hue or wave-length remains constant and the value and chroma change, as by taking a vertical plane from the axis outward. Or you can keep the value constant with the color and saturation changing by taking horizontal planes. Or you can keep the chroma constant with varying color and value by taking cylindrical surfaces at different distances from the axis.

The first grouping seems without question the best. Because,

when we see a color we always think first of its hue or wave-length. Value and chroma are qualifying factors.

The best arrangement therefore is to have all the colors of the same hue or wave-length on one chart or page. We would have then a series of such charts, as many as we wished different hues in our standard. On all of these charts the lines of cards which were on the same level from the top would have the same value. Those lines of cards which were the same distance from the sides would have the same chroma. With the three constants of a given color in mind such an arrangement enables one to find it very easily; in fact, leads one directly to it.

NOTATIONS OR NOMENCLATURE.

The color names in ordinary use are of practically no help, due to the fact that they are too general and in themselves designate no particular color, except to those who have arbitrarily learned to associate them with some particular color.

Every color has the three constants of hue, chroma and value. If any one of these constants is varied the color is varied. A color therefore can be absolutely specifically designated if its hue, chroma and value are given. Here again we are indebted to Munsell for giving us a most convenient way of doing this. The first constant, hue, he designates by a letter; for example, R for red or B for blue. The second constant, value, he designates by a number above a line to the right of the letter; the third constant, chroma, by a number below this line. For example, R 6/5 gives all three constants and determines the color. When you see such a notation you think first of red, then you notice the figure six and realize it is a little above middle value. The figure five tells you that its chroma is half-way between a neutral gray and a full chroma red. A very definite color is thus brought to your mind.

An improvement that could be made in this notation would be in the matter of the letters used to designate the hue. In a color standard there should be at least thirty or forty different hues. The words in common use to designate hue are yellow, orange, red, purple, blue and green. We can get definite hue notation if we qualify one of these words by another. That will give us all together twelve names: yellow, yellow orange, orange, etc. This, however, is not enough to cover the necessary hues. It

is possible to still further increase the number in two ways. First, by considering the last part of the combined word as the noun and the first part as the qualifying adjective and adding between yellow and yellow orange, for instance, the name "orange yellow," which would be a yellower hue than "yellow orange." After having used this notation for many years in my own system I am convinced it is very poor. When I come across it I always have to stop and think which is yellowest in hue, "yellow orange" or "orange yellow." Moreover, this adds only six more names, which does not give us enough. Second, the name could be repeated as "yellow yellow orange." But such a notation is subject to the criticism made above.

It is believed the best notation would be to use the simple and simple compound names, yellow, yellow orange, orange, etc., followed by the wave-length of the hue. The name or initial would give the general hue, the number the exact hue. The decrease in size of the number would suggest shorter wave-length and a hue slightly towards the blue end of the spectrum.

As there are no wave-lengths covering the purple region, an arbitrary set of numbers covering the least perceptible differences could be used in this region.

There is some question as to whether there is any necessity of using the letters placed before the wave-length number to designate the general hue.

The objections to the use of such letters are: that anyone using a standard would learn to know the hues by the number of their wave-length and that the letters therefore are unnecessary. That as words designating hue designate no particular wave-length, and as there are not enough of them to designate all the necessary hues, it would be better to develop the practise of thinking of hue in terms of wave-length and abandon the old names for hue.

The arguments in favor of the use of letters to designate the general hue are that the majority of persons using a color standard would not learn to know the hues by the number of their wave-length and that general designations by a symbol would be helpful, for although words designating hue do not designate particular wave-lengths, yet they do universally designate regions in the spectrum. That it would be impossible to ever develop a universal practice of thinking of hues in terms of wave-lengths. That the old names for hue are so universally accepted that they

will always be used and that it would be best to assign them to particular regions in the spectrum.

NUMBER OF CARDS.

A color standard to be of practical use must have a sufficient number of different cards, so that the color of any object can be closely matched. The degree of closeness of course varies according to what use the system is put. But for all scientific purposes, biological, for instance, and for use in mercantile business, matching textiles, arts, etc., it should be pretty close. The ideal standard set would be one in which the number of cards was such that the difference between any card and the card next to it was two least perceptible differences. The color of any object lying between cards with a separation equal to two least perceptible differences would appear like one or the other of the cards unless it lay half-way between them. Its color would therefore be definitely determined and it could be designated as lying half-way between such cards.

The least perceptible differences in the three constants under the best conditions are roughly such that we can distinguish one hundred and fifty steps in hue, two hundred steps in value, one hundred steps in chroma. The least perceptible difference between small cards such as must be used in a color standard is, however, much greater than those given above. For instance, in the set of grays which the writer made up containing sixty steps, the difference between the cards is just about perceptible. It is these larger, least perceptible, differences which should be used as the basis of the separation of a standard of cards. I should roughly estimate them to be between two and three times as great as that which you can get under the most favorable conditions. This would give from fifty to eighty steps in hue, seventy to one hundred steps in value and thirty-five to fifty steps in chroma. If the cards are separated by two such perceptible differences, we would have from twenty-five to forty steps in hue, thirty-five to fifty steps in value and sixteen to twenty-five steps in chroma. This would give a total number of about thirteen thousand different cards. It would be advisable to furnish sets with separations of three or four or more least perceptible differences between the cards, the different standards being supplied according to the use to which they would be put.

SPACING.

Whatever degree of separation is used, it should be the same throughout the entire system. That is, if the difference in value between a card and the next one above or below it was two least perceptible differences, the differences in chroma between that card and the next one to the left, or the right of it, should be the same. Also the difference in hue between the cards on one chart or page from those on the next chart or page should be the same.

The data on least perceptible differences in value or brightness have been worked out by many investigators, which makes this spacing easily determined. The least perceptible difference in hue in the spectrum have been very definitely determined for a limited number of observers by A. L. Jones.[1] It is believed, however, that it should be worked out for a large number of observers before it will be sufficiently definitely determined to use in making up a color standard.

In regard to spacing in chroma, although some data have been collected of the least perceptible differences in that constant, the best by Geissler,[2] it is not sufficient for the purpose. Further measurements would have to be made to determine the average least perceptible differences in this constant. This, however, would not be a difficult or long job, as the path has been blazed and methods established.

STANDARDIZATION.

The three constants of every card in the standard should be determined by the most accurate measurement. The best way to do this would be to first calculate in the three constants of hue, value and chroma; *i.e.*, wave-length, intensity and saturation or the amount of white light added, just what every card in the standard should be. Then these combinations should be obtained by means of a colorimeter with a standard light source in a standard illumination. Finally the cards should be made to match these predetermined combinations obtained with the colorimeter. The spectral curves of the color patches obtained by the colorimeter should be determined as should those of the various cards. If

[1] JOURNAL OPT. SOC. OF AMER., Vol. 1, Nos. 2 and 3, March–May, 1917.

[2] *Amer. Jour. of Psych.*, 1913, 24, p. 171; see also Luckiesh: Color and Its Applications (Van Nostrand), p. 127.

possible those of the cards should be made similar to those obtained by the colorimeter. The determination of the spectral curves of the cards would not be so difficult as might appear. For if, as suggested, all the cards on each page of the standard were various values and chromas of the same hue, they would be obtained by mixing white and black paint with the saturated pigment of that hue. If the spectral curve of the saturated pigment was determined, the curves of all the other cards on that page would be of the same type. This work should preferably be done by some government bureau, as the Bureau of Standards, and the colorimeter kept by them as the standard. A set of measurements of every card in the standard should accompany the standard.

Various color standards will now be considered, bearing in mind the above-mentioned requisites.

RIDGWAY'S.

Ridgway's system [3] is probably the most used to-day, and on the whole is believed to be the best system. It is defective, however, in that it satisfies but one of the aforementioned requisites of a perfect color standard.

First, *Arrangement.* It is defective in that he has grouped together in one page the fully saturated red, orange red and orange orange red, of all values from lightest to the darkest. On the next page he has red orange, orange red orange and orange, and so on around the spectrum. On later pages he has the same colors similarly grouped, but one step less in saturation. That is, on any one page he has different hues, chromas, and values; in other words, all three of the color constants are varying.

Further, different vertical lines of cards give differences in hue, but nothing in his arrangement gives differences in chroma or value. The chroma on a single sheet varies, because in mixing white or black with his pure color to get his differences in value he also gets differences in chroma in the same vertical line. And by putting all his fully saturated colors in the same horizontal line, *i.e.,* the fifth from the top, he varies his values because a full chroma yellow, for instance, is much lighter in value than a full chroma red.

Second, *Nomenclature.* The color names he uses are too

[3] "Color Standards and Color Nomenclature," Robert Ridgway, Washington, D. C., 1912.

general and do not materially help to designate the exact color. The letters and numbers he uses are subject to the following criticism: He has the hues both lettered and numbered, his lettering being complicated and his numbering arbitrary. He begins with number one for red and goes to seventy-one for violet red red. Accompanying these numbers he has letters. From orange to yellow, for instance, they run as follows: OY-O, Y-O, OY, YO-Y.OYY. This notation is clearly confusing. The different values he designates by small letters, beginning with (a) in middle value he designates the lighter values by (b), (d), (f), and the darker values by (i), (k), (m), differentiating between tints and shades. The different chromas he designates by dashes above the number for hue, one dash (′) designating one step from full saturation, two dashes (″) two steps, etc. It is evident that this is a very complicated notation.

Third, *Number of Cards.* In all he has one thousand one hundred and fifteen different cards. This number is too small. Defective arrangement and notation can be surmounted, but there is no way to make up for an insufficient number of cards.

Fourth, *Spacing.* His spacing is not even, primarily due to his faulty arrangement of putting all full chromored colors in the middle of his value scale. He gets, for instance, the same number of steps from yellow to white that he does from blue to white, and the yellow is much lighter than the blue. Even the spacing of his hues is not as even as can be determined by present scientific methods, Jones.[4]

Fifth, *Standardization.* We have Ridgway to thank for the best standardization as far as I know up to date. He based his colors on combinations from the Maxwell color disk. His data cover the spacing for all three constants and fulfill all the requirements of standardization. With the means he used it could not be improved upon. It is believed it would be much better, however, to obtain the various combinations of color constants by direct mixtures of light with a colorimeter. In the last analysis he has to base his six fundamental hues on wave-lengths. It would seem best therefore to start with actual wave-lengths, getting the various values and chromas by changing the intensity and adding white light.

[4] *Loc. cit.*

MUNSELL.

Munsell's system,[5] which has been developed since Ridgway's, is in some of the requisites better, some worse, than Ridgway's.

First, *Arrangement.* Its arrangement, which has already been described, cannot be improved upon.

Second, *Nomenclature.* His nomenclature, as far as it goes, is also, it is believed, as good as can be devised. It is wanting only in that it does not give any suggestion as to how the intermediate hues between his eleven standard ones should be designated.

Third, *Number of Cards.* As a complete color system, for which I do not believe he intended it to be, it falls down in that there are not enough different colors in the system. He only has eleven different hues, as compared with Ridgway's thirty-six.

Fourth, *Spacing.* His spacing is not the same throughout the system, that between hues being greater than those between value and chroma. His spacing between values and chromas looks very good indeed. Priest, *et al.*,[6] have shown, however, that his series of values is not in accord with the natural system expressed by Weber's law.

Fifth, *Standardization.* Munsell gives no specific account as to what measurements his colors are based upon. His system, therefore, fails in this requisite, although the examination of his system by Priest *et al.*[7] goes a long way towards meeting this deficiency.

AMES.

This standard, which is in triplicate and has never been published, was made by the writer and his sister, Mrs. Oakes Ames, before Ridgway's last edition came out and before Munsell completed his work. It was made as an assistance in painting to make exact color reproductions of scenes. The method was to match the color seen by holding up a card, noting the color on the canvas, and after the whole scene had been matched putting in the paints from the color notations. It contains twenty-seven hues, fifteen steps in value and ten steps in chroma for each hue, making in all about thirty-three hundred different cards. It fulfilled

[5] "Atlas of the Munsell Color System," Wadsworth, Howland & Co., Boston, Mass.

[6] *Technol. Papers, Bureau of Standards, No. 167,* Sept., 1920.

[7] *Loc. cit.*

every requirement expected of it. It has, however, marked defects and can be much improved upon. It will be considered in view of the five requisites of a perfect color standard.

First, *Arrangement*. The arrangement, which was in general after Munsell, cannot be improved upon, except in so far as it does not follow Munsell. That is, in the following particulars: First, all the fully saturated colors are put ten steps out from gray. For example, there are ten steps from a fully saturated red to gray and the same number from a fully saturated green to gray. Now a fully saturated red is much stronger in chroma than a fully saturated green. The arrangement, therefore, is wrong. Second, vertical lines at an equal distance from the side of the page do not designate the same chroma. This is due to the fact that in making lighter values of a saturated color we mixed white with them and kept them in the same vertical line as the saturated color, although of course their saturation was reduced by the added mixture of white.

Second, *Notation*. The notation which followed Munsell is very good, except in the designation of the hues. Names as orange yellow and yellow orange were used which, as has been said before, are confusing. The use to which the standard has been put most thoroughly tested and proved the value of Munsell's method of notation, *i.e.*, a designation for each color constant.

Third, *Number of Cards*. The total number of cards in the standard is about thirty-three hundred. In this particular it is markedly superior to the other standards. From all my experience, however, it is believed the number should be increased.

Fourth, *Spacing*. The spacing between values and chromas is about right, but there should be more hues in certain sections of the spectrum. The spacing, however, is not the same throughout the system. This is due to the poor arrangement above spoken of. The spacing in the green chart is not as great as that in the red, and the spacing in the lighter values of the red chart is not as great as it is in the lower part. If the spacings were the same throughout the system would have its same usefulness with about one-third less cards.

The spacing in value was carefully worked out with Maxwell disks, taking into account Fechner's law, and though it has not been measured, gives, it is believed, substantially even steps in sensation. The spacing in hue and chroma was done only by eye.

Fifth, *Standardization*. In this requisite the system falls down entirely. Every card can be duplicated from a tube of color which matches it, but this in no way fulfills the demands of this requisite.

The arrangement of the colors on separate cards that can be taken out of the sheets has proved to be of the greatest convenience, as it makes it possible to match surfaces that cannot be brought into juxtaposition with the different parts of the sheet.

JORGENSEN.

Jorgensen's work [8] cannot really be considered a standard for the purposes above described. He gives, however, a suggestion that might be of value in making up a color standard. That is to have the different values and chromas of each hue gradually blend into one another, the value and chroma being designated by a scale at the top and side of each chart. I imagine this would have to be done by some printing process, and I am not sufficiently conversant with the technique of color printing to know whether or not it would be possible. The great advantages of such a method, in that it would give all of the colors and in the smallest possible space and at an enormously reduced cost, seem to make it at least worth considering.

CONCLUSION.

The demand for a perfect standard color system is very great and from innumerable branches of activity, art, sciences, manufacturers, retailers.

The lines of procedure and work necessary to produce a standard system have been pretty definitely laid out.

The only reason for delay is the lack of sufficient number of individual determinations of the least perceptible differences in hue and chroma. It is most earnestly advised that these determinations be made by various laboratories and as soon as possible, in order that the great need for a proper color standard can be met.

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[8] "The Mastery of Color," C. J. Jorgensen, Milwaukee, 1906.

THE ORIENTATION OF THE GRAINS IN A DRIED PHOTOGRAPHIC EMULSION.*

BY

L. SILBERSTEIN.

A SURFACE of film base is uniformly coated with a freshly made emulsion, forming a layer of thickness b, say. The orientation of the silver halide grains which, in accordance with all available experimental evidence, will be treated here as small plates, can be assumed to be completely haphazard, *i.e.*, such that in this, the initial state of the emulsion, no directions of the plane-normals of the plates are privileged. In symbols, if θ be the angle of inclination of a grain (plate) to the film base, ranging from 0 to 90°, the number of grains whose inclination is contained between θ and $\theta + d\theta$ is

$$N\,d\theta : \frac{\pi}{2} = \frac{2}{\pi} N\,d\theta$$

N being the total number of grains. Such is, by assumption, the initial state of things.

Let now the layer, left to dry, shrink down to the thickness b' and let it be assumed that this process consists, at least macroscopically, in a *uniform contraction* of the gelatine without lateral expansion, the axis of this simplest kind of deformation being the normal to the film base.

Such being the assumptions, it is required to find the amount of orientation of the grains in the final dry state of the emulsion. More definitely, a being the average area of the microscopic plates, it is required to find the average *efficient* area per grain, say κa, *i.e.*, the average value of the projection of a upon the horizontal plane, in the final state.

The efficiency coefficient κ thus defined amounts simply to the average of $\cos \theta'$, if θ' be the final inclination of a grain-plate.

The problem is thus reduced to writing down the relation between θ' and θ and integrating over a quadrant in θ.

Now, with regard to the transformation of θ into θ', two

* Communication No. 112 from the Research Laboratory of the Eastman Kodak Company.

equally simple possibilities (both compatible with the requirement of a macroscopically uniform contraction of the medium) present themselves.

These are:

(1) $$\sin\theta' = k \,.\, \sin\theta,$$

(2) $$\tan\theta' = k \,.\, \tan\theta,$$

where $k = b'/b$ is the contraction ratio of the emulsion layer.

The first of these relations would correspond to the assumption that there is no slipping between the silver halide grains [1] and the adjacent gelatine, and that the distance x of every point of each

FIG. 1.

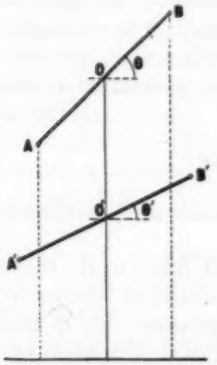


FIG. 2.

plate from the base is being reduced in the same ratio, *i.e.*, to $x' = kx$. Let $AB = l$ (Fig. 1) be a diameter of the grain-plate, *viz.* the diameter contained in the vertical plane passing through the normal to the film base, and let x_1, x_2 be the ordinates of the end-points, A, B. Then

$$\sin\theta = (x_2 - x_1) : l, \text{ and } \sin\theta' = (x_2' - x_1') : l,$$

and since

$$x_1' = k\,x_1,\ x_2' = k\,x_2,$$

we have

$$\sin\theta' = k\,\sin\theta.$$

It will be noted that this kind of displacement of the (rigid) plates implies a torsional deformation of the gelatine in minute regions

[1] Whose thickness is being neglected for the present.

around the plates in addition to the general uniform contraction. For, if a point of the plate, say its centre, describes a straight, other points of the plate, together with the adjacent gelatine particles will, under the made assumptions, describe necessarily curved paths.

The alternative relation, (2), is obtained at once if it be assumed that each particle of the medium (gelatine), such as A or B in Fig. 2, describes strictly a straight path, its ordinate being reduced in the ratio k. In fact, in this case the horizontal projection of the distance (h, Fig. 2) of two gelatine particles (A, B) remains constant, and we have $\tan\theta = (x_2 - x_1) : h$ and $\tan\theta' = (x_2' - x_1') : h = k \tan\theta$. At the same time we see that, since the gelatine filament AB is contracted to A′B′, while the originally adjacent plate diameter AB retains its length, the plate must slip or slide, in its own plane, relatively to the medium.

Which of these two possibilities (or, perhaps, other intermediate ones) takes place in reality we do not know. And it seems, therefore, advisable to consider, separately, the consequences of the relations (1) and (2).

1. By formula (1), which will be referred to as the *sine-law*, we have

$$\cos\theta' = \sqrt{1 - k^2 \sin^2\theta},$$

and therefore,

$$\kappa = \frac{2}{\pi}\int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\theta}\,.\,d\theta.$$

The integral on the right hand is the well-known (and tabulated) complete elliptic integral of the second kind, which is usually denoted by E (k); our contraction ratio k is the modulus of this integral. Thus we have the efficiency coefficient

$$\kappa = \frac{2}{\pi}\mathrm{E}(k). \qquad \text{.....(I)}$$

A short, and more than sufficiently accurate, table of E, with $\alpha = \arcsin k$ as entry, will be found, for instance, on page 68 of Jahnke and Emde's "Tafeln." We may as well use the rapidly converging power series for $\mathrm{E}(k)$ giving

$$\kappa = 1 - \left(\frac{1}{2}\right)^2 \frac{k^2}{1} - \left(\frac{1.3}{2.4}\right)^2 \frac{k^4}{3} - \left(\frac{1.3.5}{2.4.6}\right)^2 \frac{k^6}{5} - \text{.....}, \qquad \text{.....(I′)}$$

of which, in practice, the first two or three terms will give all that is required. For $k = 0$ we would have $\kappa = 1$, an obvious result, and

for $k=1$ (which is the initial state), $E(1)=1$ and $\kappa=\frac{2}{\pi}$, as it should be, this being the average of $\cos\theta$ in absence of any orientation. Thus the whole range of κ is contained between 0.6366 and unity. Thus far the thickness of the grain-plates or the breadth of their edges has been neglected. If it is taken into account and if, say, the thickness of an average grain is a fraction ϵ of the average diameter, the "efficient area" is increased by $a\,\epsilon\,k\sin\theta$, and we have the correction term $\frac{2}{\pi}\epsilon\,k$, to be added to the above value of κ. The whole reasoning is valid, of course, only when ϵ is a small fraction. In practice its value does not amount to more than 1/15.

2. Passing to the alternative assumption (2), which will be referred to as the *tangent-law*, we have

$$\cos^2\theta' = 1 : (1+\tan^2\theta') = 1 : (1+k^2\tan^2\theta),$$

so that

$$\kappa=\frac{2}{\pi}\int_0^{\pi/2}\frac{d\theta}{\sqrt{1+k^2\tan^2\theta}}=\frac{2}{\pi}\int_0^{\pi/2}\frac{\cos\theta\,d\theta}{\sqrt{1-(1-k^2)\sin^2\theta}},$$

or, substituting $u=\sin\theta$,

$$\kappa=\frac{2}{\pi}\int_0^1\frac{du}{\sqrt{1-(1-k^2)\,u^2}}.$$

The value of the integral is

$$\frac{\arcsin\sqrt{1-k^2}}{\sqrt{1-k^2}}=\frac{\arccos k}{\sqrt{1-k^2}}.$$

Thus the efficiency coefficient corresponding to the tangent-law is

$$\kappa=\frac{2\arccos k}{\pi\sqrt{1-k^2}}. \qquad \text{(II)}$$

This gives for $k=1$ and $k=0$ the values $\kappa=\frac{2}{\pi}$ and 1, respectively, coinciding, of course, with the previous ones, while for all intermediate values of k the values of κ are somewhat smaller than those given by formula (I). For contractions such as $k=1/10$ (as is usually the case in film manufacturing) the two κ-values differ only by about 6 per cent.

It is possible that in reality we have something intermediate between the conditions of (1) and (2), so that, in absence of better knowledge we can take some intermediate value between κ_I and κ_{II}, say their arithmetical mean. This becomes for such contrac-

tions as $k = 1/10$ a purely academic distinction, for then both κ – values differ but little from unity.

The following table gives, to four decimals, the values of κ on either assumption, for a few values of the contraction ratio k:

k	κ (I)	κ (II)
1.0	$\frac{2}{\pi} = 0.6366$	0.6366
0.5	0.9342	0.7698
0.3	0.9771	0.8442
0.2	0.9899	0.8899
0.1	0.9975	0.9410
0.0	1.0000	1.0000

If the average efficient area κa could be measured experimentally for various contractions k (preferably nearer to unity than to zero), we might readily decide whether in the actual drying process the behaviour of the emulsion answers better to the first or the second of our assumptions (*i.e.*, to the sine-law or the tangent-law). For the present we can say only that these theoretical results agree substantially with the observations made, for k equal to about 1/10 by Mr. Trivelli, who found that all or almost all plates (grains) lay "flat," *i.e.*, were parallel to the film base.

It may be well to make a few more remarks on the consequences of the sine-law (1) as compared with those following from the tangent-law (2).

Although, as we just saw, the difference in the κ – values, especially for low values of k, is rather small, the range of inclinations θ' in the dry plate and the distribution of the grains over this angle-range is entirely different for the two cases.

In fact, on the assumption (1), the upper limit of inclination, say α, is given by

$$\sin \alpha = k, \qquad (3)$$

so that no silver halide plate has an inclination exceeding the angle arc sin k, *i.e.*, about 5° 44′ for $k = 1/10$, for instance.[2]

On the other hand, under the assumption (2) in view of $\tan \frac{\pi}{2} = \infty$, we have for $\theta' = \text{arc tan } (k \tan \theta)$ the full range from 0 to 90°.

[2] Of course, if a grain (plate) was initially rigorously perpendicular to the base ($\theta = \frac{\pi}{2}$) there would be no tendency to tilt it, thus leaving $\theta' = 90°$; but it is scarcely necessary to say that the probability of a grain having exactly $\theta = 90°$ is zero.

Thus far the total ranges of the angles. Now for the distribution of the grains through the angle domain.

Let us denote by $N'd\theta'$ the number of grains whose inclinations in the final state are contained between θ' and $\theta + d\theta'$. Then, since all these (and these only) were originally contained within the angle interval θ to $\theta + d\theta$ and were therefore $\frac{2}{\pi} N d\theta$ in number, we have, for the determination of the unknown function N',

$$N' \, d\,\theta' = \frac{2}{\pi} N \, d\,\theta.$$

Now $\frac{d\theta}{d\theta'}$ is $\frac{\cos\theta'}{k\cos\theta}$ under the first, and $\frac{\cos^2\theta}{k\cos^2\theta'}$ under the alternative (second) assumption. Thus the number of grains whose final inclinations are contained between θ' and $\theta' + d\theta'$ is, in the case of the *sine-law*,

$$N' \, d\,\theta' = \frac{2}{\pi} N \frac{\cos\theta' \, d\,\theta'}{\sqrt{k^2 - \sin^2\theta'}}, \quad 0 \leq \theta' \leq \arcsin k, \tag{4s}$$

and in the case of the *tangent-law*, remembering that $\cos^2 = 1 : (1 + \tan^2)$,

$$N' \, d\,\theta' = \frac{2}{\pi} N \frac{k \, d\,\theta'}{k^2\cos^2\theta' + \sin^2\theta'}, \quad 0 \leq \theta' \leq \frac{\pi}{2} \tag{4t}$$

The first formula gives, for the number $n'(\theta')$ of grains whose inclinations are contained between 0 and θ',

$$n'(\theta') = \int_0^{\theta'} N' \, d\,\theta' = \frac{2}{\pi} N \int \frac{d(\sin\theta'/k)}{\sqrt{1 - \left(\frac{\sin\theta'}{k}\right)^2}},$$

i.e.,

$$n'(\theta') = \frac{2}{\pi} N \arcsin\left(\frac{\sin\theta'}{k}\right), \quad 0 \leq \theta' \leq \arcsin k, \tag{5s}$$

as it should be, for all these grains were originally contained between 0 and $\theta = \arcsin\left(\frac{\sin\theta'}{k}\right)$ of the angle domain. Similarly, the formula (4*t*) gives

$$n'(\theta') = \int_0^{\theta'} N' \, d\,\theta' = \frac{2}{\pi} N \arctan\left(\frac{\tan\theta'}{k}\right), \tag{5t}$$

valid for any θ' between 0 and 90°.

By the sine-law formula (5^s), we have, for example, for

$$\theta' = \text{arc sin } k = \alpha,$$

$$n'(\alpha) = \frac{2}{\pi} N \text{ arc sin } 1 = \frac{2}{\pi} N \cdot \frac{\pi}{2} = N,$$

that is to say, the total number of grains, as it should be, and for $\sin \theta' = \frac{1}{2} k$, for instance, we have

$$n' = \frac{2}{\pi} N \text{ arc sin} \frac{1}{2} = \frac{2}{\pi} N \cdot \frac{1}{3} \frac{\pi}{2} = \frac{1}{3} N,$$

i.e., just one-third of all grains (the remaining two-thirds being contained between arc sin $\frac{k}{2}$ and $\alpha = \text{arc sin } k$). For small k (such as 1/10), and therefore small θ', we can write formula (5^s), approximately,

$$n'(\theta') \doteq \frac{2}{\pi} N \cdot \text{arc sin} \left(\frac{\theta'}{k}\right), \quad \alpha \doteq k.$$

The tangent-law formula (5^t), which might also be obtained without integration by remembering that $\theta = \text{arc tan} \left(\frac{\tan \theta'}{k}\right)$, gives for

$$\theta' = \frac{\pi}{2}, n' \left(\frac{\pi}{2}\right) = \frac{2}{\pi} N \text{ arc tan } \infty = N,$$

as it should be. For $\theta' = 45°$, for instance, we have $\tan \theta' = 1$, and

$$n' \left(\frac{\pi}{4}\right) = \frac{2}{\pi} N \cdot \text{arc tan} \left(\frac{1}{k}\right).$$

Thus, if for example, $k = 1/10$,

$$n'(45°) = N \frac{84.3}{90} \doteq 0.94 N,$$

that is to say, 94 per cent. of the grains have their inclinations contained between 0° and 45°, and the remaining 6 per cent. between 45° and 90°.

EASTMAN KODAK CO.,
ROCHESTER, N. Y.

A METHOD OF OBTAINING RADIANT ENERGY HAVING THE VISIBLE SPECTRAL DISTRIBUTION OF A COMPLETE RADIATOR AT VERY HIGH TEMPERATURES.[1]

BY

IRWIN G. PRIEST.

THE author has previously published several detached notices and papers showing the utility of rotatory dispersion in producing radiant energy of some desired spectral distribution.[2] It is possible to obtain by this artifice spectral distributions corresponding to a complete radiator (black body) at temperatures far above anything possible in the actual operation of a furnace, or any artificial incandescent source. In certain studies in colorimetry and physiologic optics [3] it is desirable to be able to produce at will such high temperature energy distributions in the visible spectrum. In the previous papers this matter was not treated in a sufficiently unified manner, and data now at hand were not available when they were published. *The purpose of the present notice is to state the precise specifications for producing radiant energy of the visible spectral distribution of a theoretical complete radiator (by Planck's formula, $c_2 = 14350$) at any temperature between 3100° K and 7000° K, by means of an apparatus which permits of the convenient adjustment of the distribution to correspond to any temperature within this range.*[4] The detailed treatment of the theory of this method and the computations leading to these results are reserved for a subsequent paper.[5]

The source of radiant energy is a gas-filled tungsten lamp [6]

[1] Published by permission of the Director, Bureau of Standards.

The author is indebted to K. S. Gibson, H. J. McNicholas and M. C. Malamphy for assistance in some of the computations leading to the results given in this paper. The figures have been drawn by Miss Gertrude Clemens.

[2] *Phys. Rev.* (2) 6, p. 64, July, 1915; (2) 9, p. 341, Apr., 1917; (2) 10, p. 208, Aug., 1917; (2) 11, p. 502, June, 1918; (2) 14, p. 264, Sept., 1919; (2) 15, p. 538, June, 1920. J. OP. SOC. AM., Nov. 1920.

[3] *E.g.*, Troland: Trans. I. E. S., 13, p. 26, 1918. Priest: New Study of the Leucoscope, J. OP. SOC. AM., Nov., 1920.

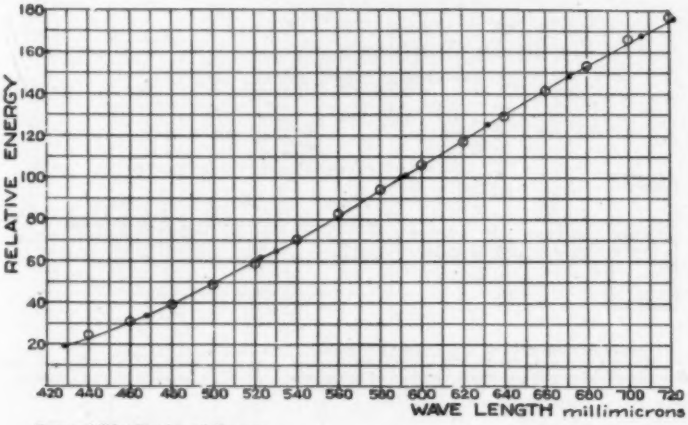
[4] Temperatures up to about 3000° K can be attained in actual incandescent sources—gas-filled tungsten lamps.

[5] The Application of Rotatory Dispersion to Colorimetry, *B. S. Sci. Paper.*

[6] B. S. Lamp 1717. The color temperature has been derived from the energy distribution curve determined radiometrically by Dr. W. W. Coblentz.

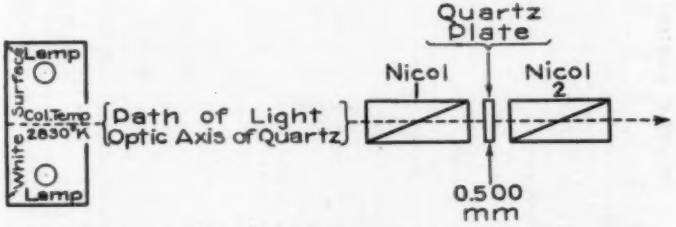
at a color temperature of 2830° K (Cf. Fig. 1). The distribution is modified by the rotatory dispersion of quartz,[1] using a quartz

FIG. 1.



Spectral Distribution of Energy.
Solid curve: Complete radiator at temperature 2830° K. by Planck's formula with $c_2 = 14350$. The small solid black circles indicate the points actually computed by Planck's formula.
Open circles: B. S. Lamp 1717 at 118.0 volts (about 15.6 l. p. w.) determined radiometrically by W. W. Coblentz.

FIG. 2.



Simple Arrangement of One Quartz Plate Between Nicols.
The spectral distribution of energy emerging from Nicol 2 in the direction of the dotted arrow may be adjusted by rotation of Nicol 2 relative to Nicol 1 (angle, ϕ).

plate 0.500 mm. thick. Two arrangements of apparatus are used, namely:

(1) The simple arrangement (Fig. 2);

(2) The duplex arrangement (Fig. 3).

The combinations of nicol prisms and quartz plates shown

[1] Cf. Priest: A New Study of the Leucoscope, Appendix I, J. OP. SOC. AM., Nov., 1920.

in these figures constitute, in effect, selective ray filters of adjustable spectral transmission. The relative transmissions as a function of wave-length are:

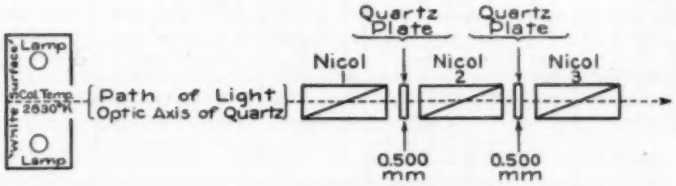
(1) For the simple arrangement,

$$\sin^2 (\phi - 0.5\ \alpha_\lambda); \quad \ldots\ldots (1)$$

(2) For the duplex arrangement,

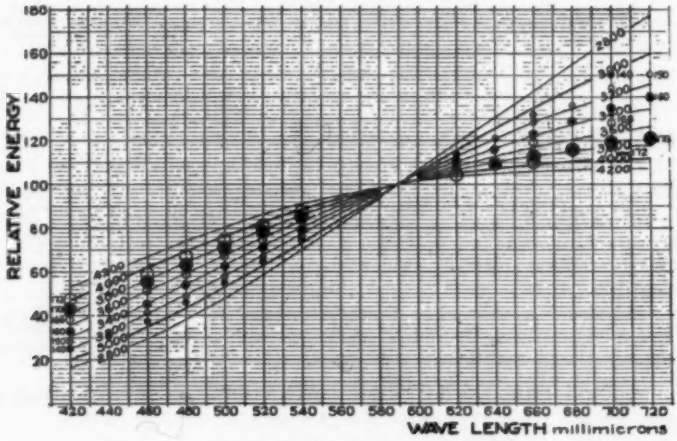
$$\sin^2 (\phi_1 - 0.5\ \alpha_\lambda) \sin^2 (\phi_2 - 0.5\ \alpha_\lambda); \quad \ldots\ldots (2)$$

FIG. 3.



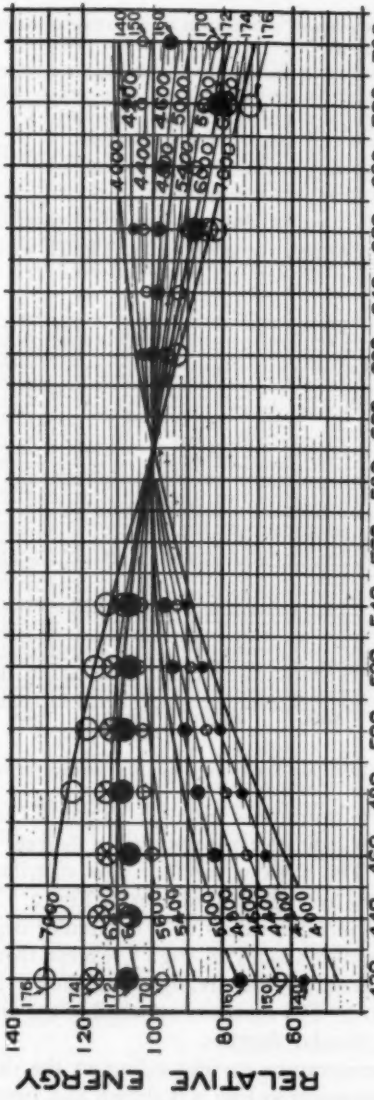
Duplex Arrangement of Two Quartz Plates and Three Nicols.
Nicols 1 and 2 are fixed. The spectral distribution of energy emerging from Nicol 3 in the direction of the dotted arrow may be adjusted by rotation of Nicol 3 relative to Nicol 2 (angle, ϕ_2).

FIG. 4.



Illustrating the Production of Radiant Energy Having the Spectral Distribution of a Complete Radiator by Means of the Simple Apparatus (Fig. 2).
The solid black curves represent Planck's formula with $c_2 = 14350$. The numbers attached to these curves indicate temperatures in degrees K.
The various circles represent distributions obtained by the arrangement shown in Fig. 2. Each different style and size of circle refers to a particular value of ϕ; and the numbers attached to the circles indicate values of ϕ in circular degrees.
In all cases, Energy = 100.0 at wave-length 590 (arbitrary convention).

FIG. 5.



Illustrating the Production of Radiant Energy Having the Spectral Distribution of a Complete Radiator by Means of the Duplex Apparatus (Fig. 2).
The solid black curves represent Planck's formula with $c_2 = 14350$. The numbers attached to these curves indicate temperatures in degrees K.
The various circles represent distributions obtained by the arrangement shown in Fig. 3. Each different style and size of circle refers to a particular value of ϕ_1, and the numbers attached to the circles indicate values of ϕ_1 in circular degrees. ϕ_1, constant = 170°.
In all cases, energy = 100.0 at wave-length 590 (arbitrary convention).

where:

$\alpha_\lambda \equiv$ rotation of the plane of polarization by 1.000 mm. of quartz for wave-length λ;

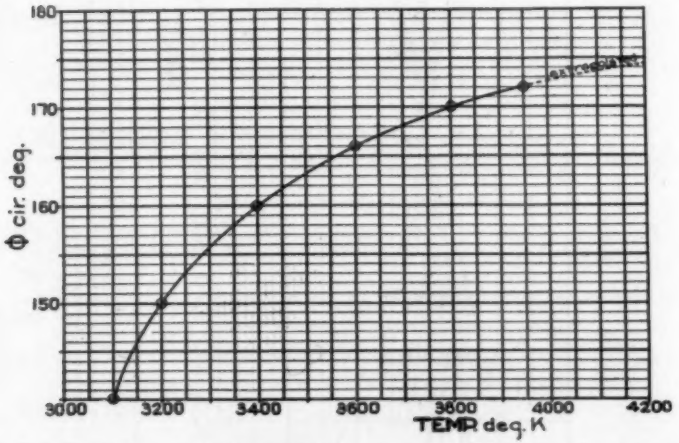
$\phi \equiv$ rotation of nicol, 2, relative to nicol, 1, in the same sense as α, from the extinction position with quartz removed (Fig. 2);

$\phi_1 \equiv$ rotation of nicol, 1, relative to nicol, 2, in the direction *contrary* to α, from extinction position;

$\phi_2 \equiv$ rotation of nicol, 3, relative to nicol, 2, with the same conventions as for ϕ above. (Fig. 3)

In Fig. 4 the solid curves show the spectral distributions of a theoretical complete radiator computed by Planck's distribution

FIG. 6.



Temperature of Energy Distribution as a Function of ϕ with the Arrangement shown in Fig. 2. (Plotted from inspection of Fig. 4.)

formula for various temperatures with $c_2 = 14350$. The numbers attached to the curves indicate temperatures in degrees K (absolute temperature). The various circles show distributions obtained by means of the arrangement shown in Fig. 2. The numbers attached to them indicate values of ϕ in circular degrees.

The same explanation applies to Fig. 5, except that it refers to distributions obtained by the arrangement shown in Fig. 3. ϕ_1 is a constant = 170°. The numbers attached to the circles indicate values of ϕ_2 in circular degrees.

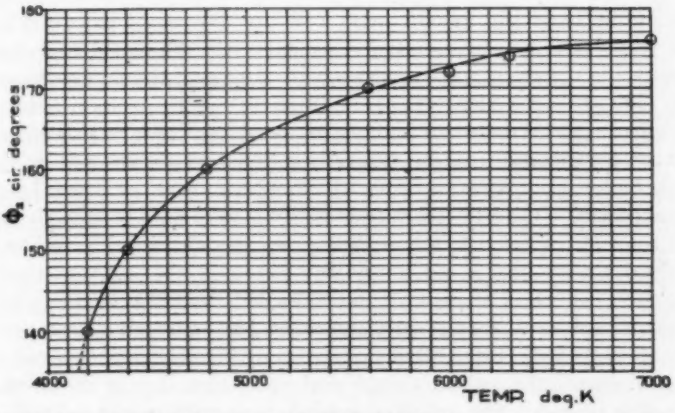
In Fig. 6 the temperature corresponding to the distribution obtained by the arrangement in Fig. 2 is shown as a function of ϕ.

Likewise, in Fig. 7, the temperature corresponding to the

distribution obtained by the arrangement shown in Fig. 3 is shown as a function of ϕ_2 (ϕ_1 being a constant = 170°).

It will be observed that very accurate representations of black-body distributions have been obtained by this method (Figs. 4 and 5). Curves drawn through each set of circles in Figs. 4 and 5 would be smooth curves very closely approximating the form of black-body curves, the only noticeable deviations occurring in the ends of the visible spectrum ($430m\mu > \lambda > 700m\mu$). Temperatures above 5000° K read as a function of ϕ_2 (Fig. 7) may be in error by about 100°; but lower temperatures read from either Fig. 6 or Fig. 7 are apparently reliable to about 10° to 20°.

FIG. 7.



Temperature of Energy Distribution as a Function of ϕ_2 (for $\phi_1 = 170°$) with the Arrangement shown in Fig. 3. (Plotted from inspection of Fig. 5.)

The Arons chromoscope [8] or similar apparatus provides a convenient means for the experimental realization of the principles outlined above. By means of such apparatus the conclusions drawn above have been experimentally verified in several cases. A determination of the spectral distribution of "normal gray light" by means of this apparatus is in progress.[9]

NATIONAL BUREAU OF STANDARDS,
WASHINGTON,
December 6, 1920.

[8] *Ann. der Phys.* (4) 33, p. 799, year 1910; and (4) 39, p. 545, year 1912.

[9] See abstract, "The Spectral Distribution of Energy Required to Evoke the Gray Sensation," elsewhere in this number of this JOURNAL.

COURSES IN OPTICS AND OPTOMETRY IN COLUMBIA UNIVERSITY.

BY

JAMES P. C. SOUTHALL.

In 1909 a law was passed regulating the practice of optometry in the State of New York, and in the following year, partly in response to the provisions of this law and at the special solicitation of the Department of Education at Albany, courses in practical optics were offered for the first time in Columbia University. Inasmuch as this new profession was to be developed on a strictly scientific basis and as optometry was properly to be regarded as a branch of applied optics, it was expressly stipulated that the academic direction and supervision of the work should be placed in charge of the Department of Physics; but the administrative control was entrusted to Extension Teaching, the great development organization of the University, through which new ideas can be tried out without being hampered too much by usages and precedents. Year by year these courses have gained steadily in favor and importance, the qualifications for admission have been raised, the attendance has continually increased, the programme of studies has been intensified and extended, and the facilities of instruction have been greatly improved and enlarged. The curriculum, as at present arranged, extends over a period of two years and demands the student's single and entire time during the whole of that period. The principal subjects are mathematics, physics, theoretical optics, general anatomy and physiology, special anatomy of the eye, theoretical and practical optometry, physiological optics and pathological conditions of the eye. Not only is this course intended to provide such special training in optics as is needed for those who are, or expect to be, optometrists in the broader professional sense, but it can be pursued with the greatest benefit by any one who has refraction work to do and who, no matter what other useful training he may have had, wishes to obtain a real knowledge of the subject and a degree of practical skill at least equal to that required of the optometrist. The entrance requirements are precisely the same as those for admission to Columbia College.

Meantime, a longer course of study has also been planned, which will take four years to complete and which is designed to give a more extensive liberal education in addition to the professional training. This new programme, which has been presented to the authorities of the University for their approval, will enable a student to obtain not only a certificate of graduation in optometry, but at the same time an academic degree corresponding to that of bachelor of science. The completion of this course will likewise satisfy the requirements for admission to the medical school; and obviously this plan affords an unusually fine foundation for a person who looks forward to specializing in diseases of the eye after having had a medical education.

When one pauses to consider how nowadays nearly everybody suffers from eye-strain and the countless ills that follow in its train and reflects that it would be hard to overestimate the inefficiency, not to say incompetence, in all the affairs of life that may be traced to the lack of suitable eye-glasses, the far-reaching importance of this kind of special training begins to be duly appreciated. Until quite recently the intellectual and scientific level of optometrical practice outside of the work of those physicians who had made a special study of it was relatively very low. The truly remarkable elevation of standards through legislation and otherwise, which has come about within the last decade or more, is due in large measure to the single-hearted devotion and efforts of a few outstanding leaders among the optometrists. Often these have been very capable and gifted men with high ideals; and here and there some of these pioneers have made notable scientific contributions of recognized value and utility. The spiritual disciples of these men, fired with youthful enthusiasm for their new work, are eager at present to prove their skill and make further advances. According to this view optometry is a profession which, though still in process of development, and in spite of its comparatively humble origins and environment, is rapidly forging ahead and gaining recognition and prestige. Undoubtedly, it is a movement that both deserves encouragement and needs wise guidance. If optometry in the future is to take rank among the learned professions and is to enlist among its votaries the highest types of trained students, it must be taught systematically in the colleges. The fact that scientific training in refraction-work is now being given in at least two of the great universities of this

country indicates that real progress is being made in this direction. It is in this field more than anywhere else that applied optics becomes a matter of vital interest in the large question of human welfare. It may be added also that although optics is one of the oldest of the sciences and its modern applications are very manifold, nevertheless spectacle glasses may be said to have been its only outcome of worldwide commercial importance. This statement needs to be emphasized, because in considering the larger aspect of the establishment in this country of a school of optical engineering and the scientific development of the optical industries in all their various branches, I think it is well perhaps to see clearly that the heart and nucleus of all projects of this kind are to be found in such beginnings as have been made in Columbia University in connection with the courses in Optics and Optometry.

In order to appreciate rightly the importance of this work, it should be viewed in its historical perspective; and for that purpose a brief survey of the development of modern ophthalmology will probably not be without interest here. Some of the statements in this sketch will be found to be quoted more or less freely from the current *Announcement of the Courses in Optics and Optometry.*

The oldest applications of optics were undoubtedly for the purpose of aiding vision, and the history of spectacle glasses dates back to about the time of Roger Bacon in the thirteenth century (or "Frier" Bacon, as he is called in Robert Smith's "Compleat System of Opticks"); although whether they were first invented in England or in Italy is one of those obscure questions which will probably never be answered. The late Professor Silvanus P. Thompson has called attention to a portrait of Pope Leo X, painted by Raphael in 1517, in which that pontiff is represented as wearing spectacles; but at the same time Professor Thompson reminds us that there is likewise a picture in one of the churches in Florence in which St. Jerome, who flourished as far back as the fourth century, is depicted in his cell with a pair of spectacles beside him; which is just one of those anachronisms which are so common in mediæval art.

Not until towards the latter part of the eighteenth century was there any really noteworthy progress in the art of making spectacles. Benjamin Franklin, as might almost be guessed, was the first to use bifocal glasses, and Sir David Brewster (whose remarkable and acute insight into all the problems of practical

optics seems, so to speak, uncanny!) proposed and worked out multifocal lenses, which, however, have never had any wide application. "Periscopic" lenses, indeed quite similar to the glasses which are worn nowadays, were recommended by Wollaston in a notable paper which attracted little attention in his own day. Meanwhile, the foundations of physiological optics had been laid by Christopher Scheiner, whose celebrated treatise entitled "Das Auge oder die Grundlage der Optik" was published in 1619, and also by DesCartes. Many years later the versatile Thomas Young found himself irresistibly drawn to the same fertile but uncultivated fields of research; and incidentally it was he who in 1801 first recognized the existence, not to say prevalence, of astigmatism in the human eye. About a quarter of a century afterwards Sir George Airy corrected this error in his own eye by means of a cylindrical lens. From this time onwards progress in ophthalmology has been unretarded. Probably no scientific labors have ever been attended with greater blessings to mankind than the brilliant work in this vast new territory which is associated especially with the names of Donders and von Helmholtz along about the middle of the nineteenth century. To the student of science it is interesting to note how the training of both Young and von Helmholtz, each of whom, starting out at first on a medical career, afterwards gained his great renown as a physicist, peculiarly qualified them to investigate and chart the unexplored regions of physiological optics, which remains to this day very much as those great pioneers left it. Their horizons were not bounded by the narrow domains of a single science, as is too often the case with more recent investigators of these obscure phenomena.

Coming down nearer to our own time, we all know of the extraordinary modern developments in all directions in both ophthalmology and applied optics, particularly in Germany and other foreign countries. Of late years England is showing signs of resuming her rightful place in optical science which she had in the days of Newton and his successors. Few persons are perhaps aware of the rapid rate at which spectacle optics is developing into a severe scientific pursuit. It would be a very superficial view to suppose that modern improvements in the manufacture of spectacles consisted chiefly in the introduction of lighter and more adaptable frames and of new and ingenious processes of fusing bifocal lenses or of grinding toric surfaces; although these and

other mechanical refinements are by no means insignificant achievements. This would be quite a mistake. It may be that the teaching in the schools has remained much as it was; but, as has been already intimated, the training of spectacle opticians to deal with the problems of astigmatism not only in the eye itself, but in the correction-glass, and with many similar problems, has made great strides and under the stimulus of the recent optometry legislation both here and abroad is assuming an important development. Improved clinical methods and apparatus are in evidence, new optical journals are being started, and the young practitioners are eager to be in the forefront. It would be difficult to overestimate the importance of the advances in ophthalmology which are to be traced to the work of Allvar Gullstrand, in Sweden, who, in collaboration with the disciples of Abbe, in Jena, has made contributions, both by his writings and by his optical appliances, which are quite worthy to be compared with those of von Helmholtz himself. As long ago as 1914 Carl Zeiss had made a new ophthalmoscope according to Gullstrand's optical specifications, which for the examination and diagnosis of diseased conditions of the eye is a marvel of instrumental perfection; although, alas! this instrument is almost unknown in this country. By means of telescopic spectacles designed by von Rohr thousands of old soldiers whose eyesight was almost ruined in the war are now being enabled to pursue useful employments.

The main purport of this argument is to set forth as simply as possible the ideals which we have had in mind in the development of the courses in Optics and Optometry in Columbia University; and while we do not pretend to have attained as yet anything beyond an imperfect and partial realization of some of our ideals, we believe profoundly that year by year we are making steady progress towards our goal. The two-year curriculum which we have at present is probably as nearly satisfactory as it can be, and it would be hardly practicable for a student who is not presumed to have had any previous college training whatever to acquire within the same length of time a deeper insight into the fundamental problems of spectacle optics and vision than is intended by this programme of studies. The fact is, it takes a boy or girl of rather exceptional ability to complete it in the minimum period of two years. The entire structure is reared on a basis of physics and elementary mathematics, which are two of the

principal subjects in the first year; but at the same time from the very outset the pupil begins to take up other subjects which bear more directly on his vocation; for example, theoretical optics, laboratory practice, shop-work and general anatomy, with special reference to the anatomy and physiology of the eye. Naturally, when he comes to his second year, his work becomes more specialized and more practical; and although he still continues to devote a considerable portion of time to theoretical optics, and particularly to the theory of spectacle glasses, now he enters also on the study of physiological optics, and at the same time he begins to acquire the more distinctly professional part of his training; for example, in such courses as theoretical and practical optometry, where he is taught systematically and thoroughly all the various processes of optometry and the most approved modern methods of sight-testing and refraction. In the latter half of this year, beginning early in February and continuing until the end of May, an optometrical clinic is conducted every Monday and Thursday afternoon. This clinic is free to the public, and the students themselves act as assistants to the instructors in charge. A preliminary examination of each patient is made by one of the students in attendance, which is then checked and criticized by the members of the clinical staff. Two examination rooms have been furnished and equipped with every necessary appliance for making these tests. In addition to obtaining this clinical experience, the student has also laboratory courses in physics, lens-measurements and physiological optics; and during the whole of his second year he is required to spend four hours each week in shop-work, with a view to acquiring skill in all the ordinary operations of mechanical optics (grinding, surfacing, centering, edging, neutralizing, and transposing lenses, bridge-bending, frame-fitting, etc.). Thanks to the generous and helpful coöperation of the American Optical Company, the Bausch & Lomb Optical Company, the Standard Optical Company, the General Optical Company, the Globe Optical Company, and others, the laboratories and shops are equipped with all the most modern optical instruments, including clinical appliances of all kinds, and with the latest types of machines for grinding, surfacing, edging, drilling, cutting, etc.; so that the facilities for work in all these manifold directions are probably quite unsurpassed. Moreover, during the whole of the second year the pupil is given a special course in pathological conditions of the eye, which is

intended chiefly to show him where the territory of the optometrist ends and that of the physician and surgeon begins, and to familiarize him to some extent with those numerous and complex cases of eye-trouble which, as a mere refractionist, it is beyond his power to do more than ameliorate, if indeed he can do even so much as that. Here he is constantly reminded of the limitations of his powers and taught that in such cases it is his duty to send the patient to some other specialist.

As might be supposed, already there is beginning to be a demand for postgraduate courses in optometry, and either something of that kind or perhaps the extension of the present curriculum so that it will require three years for completion instead of two is the next step to be considered. And here, it must be confessed, we are now more or less at sea, because beyond this point it is all uncharted.

After all, as was said in the beginning, optometry is but one branch of applied optics. Already in England a significant effect of the increased interest in the study of spectacle optics is to be seen in the establishment of one or more schools of optical engineering, where it is now possible for a student to pursue extensive studies in optics, both theoretical and applied, including courses not only in such subjects as physiological optics and the making, testing and fitting of spectacle lenses, but also in the principles, design and construction of optical instruments in the arts of both war and peace, and where presumably optical researches of all kinds are to be systematically fostered and encouraged. As I write, I have before me a prospectus of two courses of lectures on designing and computing of lens-systems which were given first in 1917 by Professor Conrady in the Department of Technical Optics in the Imperial College of Science and Technology at South Kensington. But it was the great war more than anything else that roused England at last to a realization of her plight, due to the long neglect of the optical sciences and industries. Professor Cheshire, writing, recently in *Nature* (civ., 1920, pages 530, 531) on "The Outlook of British Technical Optics," says:

> "We shiver yet when we remember the single thread upon which the production of optical munitions depended in this country. Our glass-makers, beaten by their foreign rivals, receiving neither help, encouragement, nor even recognition from the Government, had been content to continue their patriotic efforts to maintain the industry, on the urgent representations of a few far-seeing scientific men, until long after those efforts held out any promise of pecuniary reward.

That danger, happily, has passed, and the complete solution of the optical glass problem is now only a question of time. Many of the glasses now produced in this country compare favourably with the best of those of our foreign rivals. The varieties available are limited, but the leeway is being rapidly made up.

"It is often stated that the late supremacy of the Germans in optical production was the direct and necessary result of the glass-making labours of Abbe and Schott, completed in the year 1886. This is not a correct statement of the case. The fact is that, when Abbe and Schott broke down the barriers to optical progress imposed by the limited varieties of glass available, Germany had in reserve a small army of scientific workers, equipped with the necessary technical knowledge and skill, ready to fill the breach and carry on the work of utilising the new glasses in the invention of new optical systems and in the improvement of old. But the world moves quickly, and inventions and discoveries, however valuable intrinsically, are likely to remain barren unless a country has a sufficient number of men equipped with the necessary knowledge to exploit them instantly and to the full. Indeed, it is only such men that can appreciate the value of inventions and discoveries. The necessity for a broad and generous scheme of national education in optical matters thus becomes apparent. When the users of optical instruments are sufficiently educated to be able to distinguish and appraise good designs and work, makers will be encouraged to meet their demands. In the absence of such education the faddist has his day, and the maker concerns himself too often in meeting the demands of fashion.

"It is satisfactory to know, then, that, so far as this country is concerned, a great deal has already been done to foster optical education. The establishment of the Technical Optics Committee, which includes representatives of the British Optical Instrument Makers' Association, the War Office, the Admiralty, the National Physical Laboratory, the London County Council, the Royal Society, and the Imperial College of Science, is in itself sufficient evidence that the question has been taken up with great thoroughness. The establishment of a department of optical engineering and applied optics at the Imperial College will ensure a supply of capable and well-educated young men for the needs of the industry generally. Prof. Conrady is doing yeoman service in the establishment of an English school of optical designers and computers, the need for which was so acutely felt during the war. The outlook, then, so far as education is concerned, is decidedly promising. Indeed, in some important respects the scheme of education here is already in advance of that of any other country."

During the war also an Institute of Applied Optics was founded in France. The studies which should be included in a school of this kind are, for example, geometrical optics, physical optics, physiological optics, optometry, theory of modern optical instruments, lens design and lens testing, manufacture of optical glass, refractometry, photometry and illuminating engineering, spectrophotometry, colorimetry, etc. Courses corresponding to some of these mentioned above are already being given in Columbia University, and the writer himself is strongly of the opinion

that this university with its commanding position and unequalled facilities offers by far the best place in the United States for the establishment of a school of optical engineering, especially in conjunction with the courses in optics and optometry which are now being carried on there; and with a modicum of encouragement from outside, he has reason to hope that the authorities of the University would view such a project with favor.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY,
November 15, 1920.

THE ANGULAR DEFLECTIONS PRODUCED ON TRANSMITTED LIGHT RAYS BY SLIGHTLY INCORRECT INTERFACIAL ANGLES OF REFLECTING PRISMS.

BY

F. E. WRIGHT.

In the manufacture of precision optics a tolerance of $\pm$ 2 minutes of arc is considered a fair and reasonable specification for most prism angles. In certain prisms, such as the Dove erecting prism and the penta prism, the angle must be correct within $\pm$ 1′ of arc; in the roof-angle prism, the roof-angle must be correct within a few seconds of arc. These prism-angle tolerances are commonly measured optically and the specifications are stated in terms of permissible angular deflections of transmitted rays from their prescribed paths.

In conformity with the practice adopted at Frankford Arsenal on the recommendation of Col. G. F. Jenks, in 1910, for the inspection of the optical performance of prisms, the following conventions may well be established and followed: A reflecting prism in an optical instrument serves to change the direction of the optical axis of the instrument through a definite, prescribed angle. Let the plane in which this angle is measured be called the "axial plane." Then the "axial-angle error" of a prism placed with its reflecting surface in the correct position is the angular component, in the axial plane, of the deviation of the reflected ray from its prescribed path after emergence from the prism. The "side-angle error" is the angular component, in the plane normal to the axial plane, of the deviation of the reflected ray from its prescribed direction after emergence from the prism. The Dove erecting prism serves only to invert and not to change the direction of the axial rays after emergence; for this prism, which presents a limiting case, let the axial plane be perpendicular to the hypothenuse surface and to the first surface entered by the incident rays; it is the plane of incidence of the incident rays and contains the normals to the incident refracting surface and to the hypothenuse surface. These conventions establish the reflecting surface of a prism as the surface of reference. Any other surface might be used, but

for the sake of uniformity in the testing and designation of the different types of prisms the reflecting surface appears to serve the purpose best. In the case of roof-edge prisms and penta prisms two reflecting surfaces replace the single reflecting surface and serve to invert the image; in these types of prisms the two reflecting prism-faces function as a unit and may be treated as such.

Experience has shown that different optical methods of different degrees of sensitiveness may be used to test the optical performance of any given type of reflecting prism. The deviations should be measured of those rays which traverse the prisms along the path which they follow in the actual instrument; in other words, a measure of the optical performance of the prism should be taken.

In view of the fact that the path of a ray through a prism depends on a number of different factors, such as defects in the glass, degree of flatness of prism surfaces in addition to different interfacial prism angles, the only feasible method for ascertaining the deviations of transmitted axial rays from the prescribed path, which result from slight departures of the several prism angles from the prescribed angles, is to consider the glass to be optically homogeneous and free from defects and the prism surfaces to be optically flat. Under these assumptions it is possible to ascertain the competency of the several different optical methods available for testing and to determine the significance, in terms of actual prism angles, of different optical tolerances which may be set. The relations between actual prism angles and the resulting ray deflections in the axial and side (normal to axial) planes are indicated for the different types of prisms in Figs. 1 to 7. In these figures a section of the correct prism is indicated by dotted lines; the incorrectly oriented surfaces are indicated by full lines; the paths of the rays are indicated by full lines. In each set of curves the ordinates are the angles of departure t of the prism surface from which the ray emerges; the abscissæ the deviations d of the emergent ray from the prescribed path; the sets of diagonal lines represent the departure angles s of the incident prism surface.

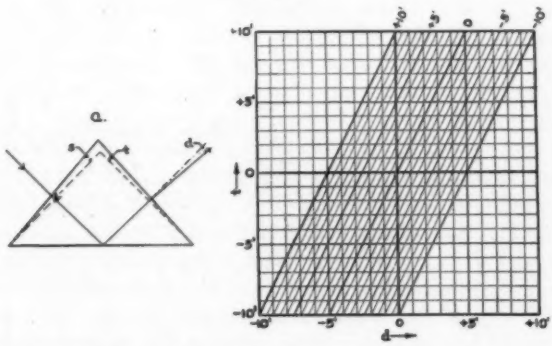
THE RIGHT-ANGLE REFLECTING PRISM.

Three optical methods may be used to test this type of prism; in two of the methods a telescope and a distant target or collimator are employed; in the third method an autocollimator serves the

purpose. A variation of these methods is obtained by the use of a projection screen in place of the observing telescope.

FIRST METHOD. *Axial-angle Errors.*—Let it be assumed, for the sake of simplicity, in this and other cases of axial-angle error, that the faces of the prism are in the same zone; this is practically the case in all prisms and the slight departures from this condition introduce departures of the second order only in the axial-angle errors. Let s be the error of the angle (positive if the total angle exceeds 45°), which the first side includes with the hypothenuse (Fig. 1*a*) and t the error of the second side face. These errors are always small and do not exceed 10 minutes, even

FIG. 1.



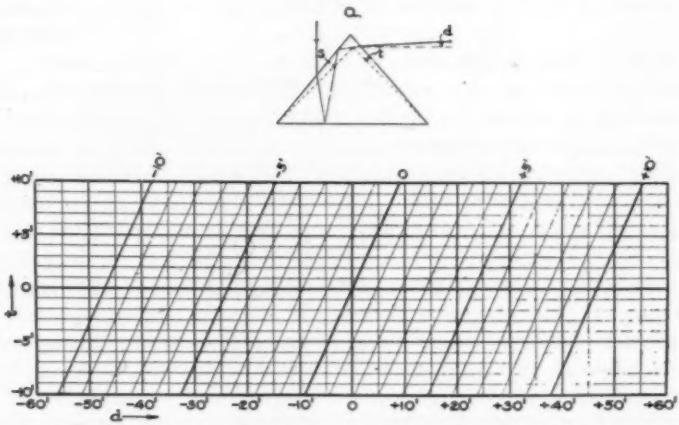
In this diagram are given the deviations of the emergent ray from its prescribed path after transmission through a right angle prism of slightly incorrect interfacial angles, as indicated in Fig. 1(*a*).

in very poor work. Let the direction of the incident ray include an angle of 45° with the reflecting face of the prism. The deviation of the emergent ray from its prescribed path in the axial plane is then $d = (n-1)(t-s)$ wherein $n = 1.515$ is the refractive index of the prism. In Fig. 1 the deviations of the rays are represented by the abscissæ, the angles t by the ordinates and the angles s by the series of oblique lines; thus a prism for which the first interfacial angle is 45° 02′ ($s = +2'$) and the second interfacial angle is 45° 01′ ($t = +1'$) causes a deviation of the emergent ray in the axial plane of $-\frac{1}{2}'$ from the prescribed path.

Side-angle Errors.—In this and other cases of side-angle error the angles which the side faces include with the hypothenuse

face are considered to be exactly 45°. This assumption is made in order to facilitate the computation, and the error introduced by it is of the second order only because the departures in these angles from 45° are very slight. In this method the incident ray enters along the normal to the first side surface, is reflected at the hypothenuse face, and is then refracted at the second side surface whose trace on the hypothenuse face includes a small angle t with the trace of the first side face on the hypothenuse face. Under these conditions the deviation of the emergent ray for a

FIG. 2.



Constant deviation method. In this figure are shown the deflections of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 2(a).

prism of refractive index, $n = 1.515$, is $d = 1.07 \cdot t$; in other words, the deviation is practically equal to the departure angle t.

With this method, therefore, the angular deviation of the emergent ray from the prescribed path, both in the axial plane and the side plane, is of the same order of magnitude as that of the errors in the interfacial angles, and the method is not especially sensitive. The method has the advantage, however, that the prism is tested under conditions resembling those of actual use.

For routine testing by this method, either a spectrometer or a combination of a telescope of adequate resolving power and a collimator or telescope and distant target is used. The prism is

placed with the hypothenuse surface against a fixture having rounded bearing points for both the hypothenuse and first side surface. The fixture is accurately adjusted so that the plane of incidence contains the normals to the hypothenuse face and to the first side face. A coördinate scale in the image plane of the telescope indicates the axial-angle and the side-angle departures from the prescribed path. In place of the observing telescope a projection screen with properly arranged coördinate lines to mark the tolerance limits may also be used.

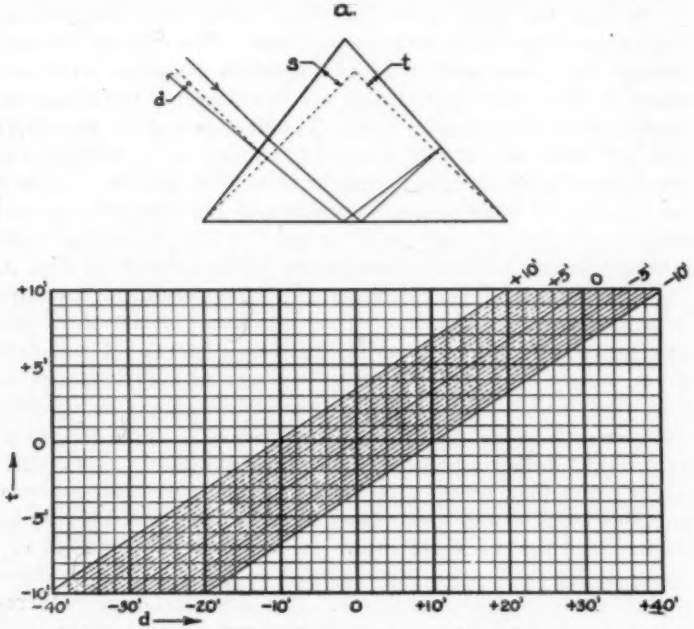
SECOND METHOD. THE CONSTANT DEVIATION METHOD OF FRANKFORD ARSENAL. *Axial-angle Error.*—The paths of the rays through the prism when used as a constant deviation prism are shown in Fig. 2*a*. The incident ray is considered to be perpendicular to the hypothenuse face. The deviation d, of the emergent ray from 90°, is then $d = 2s + (i - r) - (i' - r')$, wherein i is the angle of incidence, r the angle of refraction, and $\sin i = n \sin r$ $(n = 1.515)$; r' is the angle of incidence of the emergent ray and equal to $(r - 3s - t)$, and $\sin i' = n \sin r'$. The deviations d for different values of t and s are shown by the curves of Fig. 2. From these curves it is evident that in a prism whose angles differ from the correct angles by $\pm 2'$, the maximal deviation of the emergent ray from the prescribed path may be 11′. The curves show, moreover, that if the prism be observed only in one position, this method may not be adequate to determine its quality; thus for $s = 0'$ and $t = +10'$ the deviation is $+9'$; if now the prism be reversed so that $s = 10'$ and $t = 0'$ the deviation $d = 46.5'$. It is thus evident that, unless care is taken to measure a prism from both directions, erroneous conclusions regarding its quality may be drawn from the result obtained. If, moreover, the incident ray enters at an angle other than normal to the hypothenuse face, slightly different deviations result. The deviation depends therefore not only on the prism angle errors, but also on the position of the prism during the observation. The last uncertainty is eliminated by prescribing that the incident ray shall coincide in direction with the normal to the hypothenuse face.

Side-angle Error.—Let the prism side-angle be 45°; let the direction of the incident ray coincide with the normal to the hypothenuse face; let the plane of incidence include the normals to the first side face and to the hypothenuse face; let t be the angle between the traces of the first side face and of the second side

face, respectively, on the hypothenuse face. The deviation d of the emergent ray from the prescribed path is then approximately $t/2.1$ for $n = 1.515$; in other words, the deviation is about half that of the angle t and the method is not especially sensitive in this respect.

THIRD METHOD. AUTOCOLLIMATION METHODS. *Axial-angle Error.*—In these methods a well corrected autocollimator of

FIG. 3.

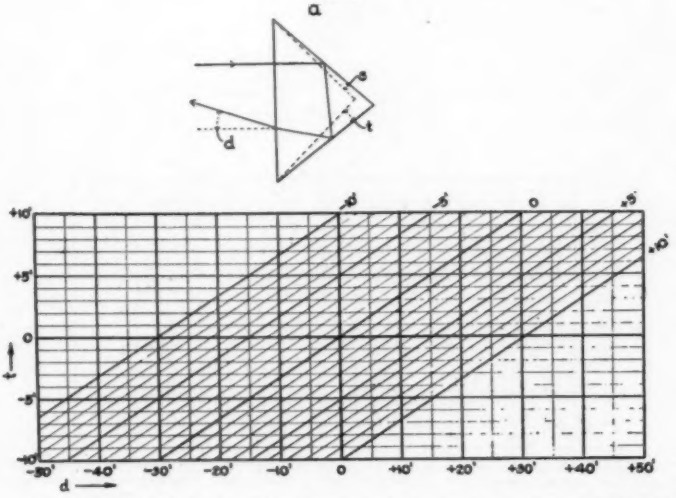


Autocollimation method. In this diagram are shown the deviations of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 3(*a*).

adequate resolving power is used; the hypothenuse surface is placed at an angle of 45° with the axis of the collimator (Fig. 3*a*). The deviation of the emergent ray from the original line of sight is then $d = 2nt - 2(n-1)s = 3.03\,t - 1.03\,s$ for a prism of refractive index $n = 1.515$. The curves of Fig. 3 show that for errors in the prism interfacial angles of $s = +3'$ and $t = +1'$ the deviation

$d = 0$; for $s = -3'$ and $t = +1'$ the deviation is $d = +6'$. The method alone is therefore not adequate to test the prism angles. If, however, the prism be rotated and the hypothenuse face be placed normal to and facing the collimator (Fig. 4*a*), the curves of Fig. 4 show that the deviation, as computed from the formula $d = 2n(t + s)$, for $s = 3'$ and $t = +1'$ is $d = +12'$; $s = -3'$ and $t = +1'$ the deviation $d = -6'$. The measurement of the deviations of the prism in these two positions suffices therefore to measure the angles satisfactorily. The autocollimation method is much more

FIG. 4.



Autocollimation method. A graphical representation is given in this diagram of the deviations of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 4 (*a*).

sensitive than the first method and is less encumbered with defects than is the second method.

Side-angle Error.—With the prism placed in the position of Fig. 4*a* the side-angle deviation is practically the same as that obtained in the first method, namely, $d = 1.07 \cdot t$, or $d = t$ approximately.

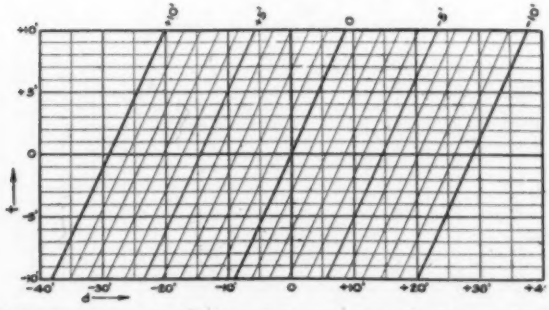
A projection method may also be used with autocollimation, but this method is simply a modification of the telescope methods and involves nothing new in principle.

THE DOVE ERECTING PRISM.

Axial-angle Error.—This prism is used to invert the image and not to change the direction of an axial ray in the telescope. In the method commonly used for testing this prism a collimator or a distant target is employed, together with an observing telescope with coördinate scale in its image plane. The prism is placed in front of the telescope with the hypothenuse face parallel with the line of sight. The path of the axial ray through the prism is shown in Fig. 5*a*. The deviation d for prism interfacial angle errors s and t is $d = 45° - t - i'$ wherein $\sin i' = n \sin r'$;

FIG. 5.

a



In this figure are shown the deviations of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 5(*a*).

$r' = r + s - t$; $\sin (45° - s) = n \sin r$ and $n = 1.515$. The curves of Fig. 5 show that for $s = +1'$ and $t = -1'$, $d = -3.8'$; for $s = +3'$ and $t = +10'$; $d = 0$; but if the prism be reversed so that $s = +10'$ and $t = +3'$, the deviation $d = -26'$. In testing a prism by this method, therefore, it should be examined in both positions, otherwise serious errors may arise. This fact should also be noted in the assembly of the prism.

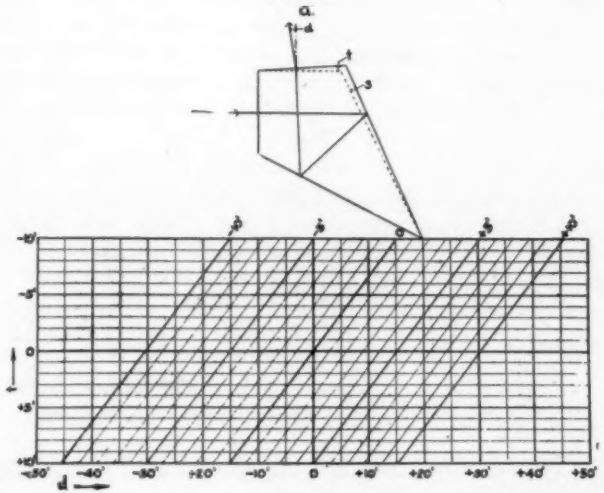
Side-angle Error.—On the assumption that the side-angles are correct and equal to 45° and that the axial plane is normal to both the hypothenuse and first side faces, the trace of the second side surface on the hypothenuse face includes a small angle t with

the trace of the first side face on the hypothenuse face, the resulting side-angle deviation of a transmitted axial ray is approximately $d = -t/2.1$ for $n = 1.515$.

THE ROOF-EDGE PRISM.

This prism may be treated as a simple reflecting prism in which the hypothenuse reflecting surface has been replaced by two reflecting surfaces mutually at right angles and truncating, like a roof, the elongated edges of the hypothenuse face. The roof-angle

FIG. 6.



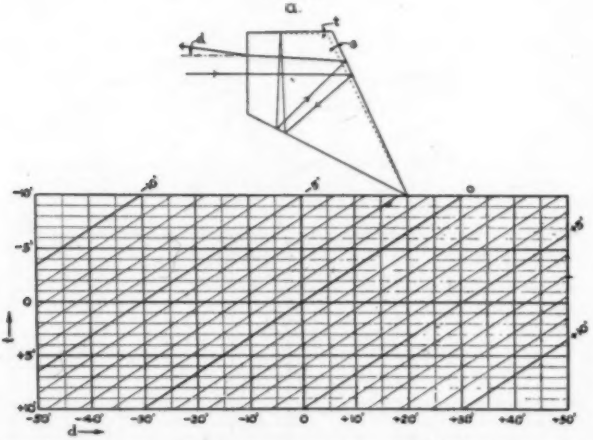
In this diagram are represented the deviations of the emergent ray from its prescribed path after transmission through a penta prism of slightly incorrect interfacial prism angles, as indicated in Fig. 6(a).

must be exactly 90° within a few seconds of arc, otherwise a double image results. The incident light strikes the first reflecting surface at an angle of 120°, is reflected across to the second face, and is reflected by it at an angle of 120°, thus inverting the image completely. The tests for axial-angle and side-angle errors may be made by the methods used for ordinary reflecting prisms; of these the autocollimating methods appear to be the most satisfactory. The correctness of the roof-angle is determined by the absence of a double image of the lines on the horizontal line of the

target. The following rule is useful in determining, in the case of a double image, whether the roof-angle is greater or less than 90°. Rack the eyepiece of the observing telescope inward toward the prism; if the two images approach each other the roof angle is less than 90°; if they approach when the eyepiece is racked outward the roof angle is greater than 90°. Another method for testing the roof-angle is to grind and polish a small face truncating the edge between the two side faces of the prism; a ray of incident light normal to this face should be reflected by the roof faces as a

FIG. 7.

a.



Autocollimation method. In this diagram are shown the deviations of the emergent ray from its prescribed path after transmission through a penta prism of slightly incorrect interfacial angles, as indicated in Fig. 7(a).

single ray parallel with the original direction. This test is made with an autocollimator; the method is, however, hardly to be recommended, as it offers no advantage over the foregoing method and requires the preparation of an additional face on the prism.

THE PENTA PRISM.

Methods similar to those described in the foregoing paragraphs are used for testing penta prisms optically. In all cases the prism should be held against a proper fixture so that the incident ray enters normally to the first surface of the prism as indicated in Figs. 6 and 7.

FIRST METHOD.—A collimator or distant target with horizontal and vertical lines is used in conjunction with an observing telescope or projection screen.

Axial-angle Errors.—For a ray entering the prism normal to the first face as shown in Fig. 6*a*, the deviation of the emergent ray from the prescribed path is $d = n(2s - t)$ wherein t and s are the small angular errors in the interfacial angles of the penta prism, and $n = 1.515$ is its refractive index. The series of curves in Fig. 6 are a graphical solution of the foregoing equation; thus for $t = +2'$ and $s = -1'$, $d = -6'$; for $t = +2'$, $s = +1'$, $d = 0$. The equation shows, moreover, that a slight change (s) in the angle between the two reflecting faces has an effect in deflecting a transmitted ray twice as great as that produced by the same change (t) in the angle between the other two faces. The reflecting faces here function as a unit just as do the reflecting faces of a roof-edge prism.

Side-angle Error.—Let the line of intersection of the two reflecting faces include an angle t with the line of intersection of the other two faces. This gives a twist to the prism. Let the angle between reflecting faces be 45° and that between the other two faces 90°. Then the angle of deflection is approximately equal to $d = n \cdot t$ wherein $n = 1.515$, the refractive index of the glass.

SECOND METHOD. AUTOCOLLIMATION METHOD. *Axial-angle Error.*—The deflection in this case (Fig. 7) is $d = 2n\ (2s - t)$ or twice that obtained by the first method; thus for $t = 2'$ and $s = -1'$, $d = -12'$; for $t = 2'$ and $s = 1'$, $d = 0$. Similarly the side-angle error is approximately twice that found by the first method. The results prove, however, that in these methods a compensation may take place whereby zero deflection may be obtained from a prism whose angles are incorrect.

SUMMARY.

Different methods are available for the testing of the interfacial angles of a reflecting prism of any given type. The angles may be measured directly on a goniometer or a spectrometer or by means of delicate gauges or of specially mounted test plates, such as the optical square. They may also be measured by the deflections produced by the prism on transmitted rays of light. A prism of the prescribed shape deflects the transmitted rays along a prescribed path. A prism of slightly incorrect interfacial prism

angles deflects transmitted light rays so that these emerge along a direction slightly different from that prescribed; from this deviation of the emergent rays of light from the prescribed path it is possible to determine the degree of exactness of the interfacial prism angles. Methods of test based on this principle enable the observer at the same time to draw conclusions regarding the quality of glass in the prism and the degree of flatness of the prism faces; in short, to judge of the fitness of the prism as a component of any given optical system. In the present paper prisms of different types are considered and different sets of curves are shown in diagrams illustrating the deviations of transmitted light rays from the prescribed paths caused by slightly incorrect interfacial prism angles. From these curves the sensitiveness of the several optical methods which have been proposed for testing the accuracy of prism angles by the use of transmitted light rays can be determined directly and the suitability of each method thereby ascertained.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
January, 1921.

THE SPECTRAL DISTRIBUTION OF ENERGY REQUIRED TO EVOKE THE GRAY SENSATION.*

[ABSTRACT.]

BY

IRWIN G. PRIEST.

General Nature of Paper.—The chief significance of this paper lies in the development and testing of an experimental method for determining an objective physical standard of white light.

Common experience teaches that the color evoked by the light from an incandescent solid (oil and gas flames, electric incandescent lamps, etc.,) depends upon the temperature of the source. Sources at comparatively low temperatures appear red; as the temperature rises the color becomes yellow of paler and paler tint until, for the highest temperatures at which artificial sources can be operated, it approximates to white. Still, all artificial incandescent sources appear yellow relative to the sun. On the other hand, some stars appear blue relative to the sun. Following Ives, Troland and others, the basic concept of this paper is this: Light conforming to the Planckian distribution formula for some particular value of temperature should evoke the hueless sensation of brilliance commonly called "white" or "gray," and recognizable neither as bluish nor yellowish.

Specific Purposes of Investigation.—The purposes of this investigation have been:

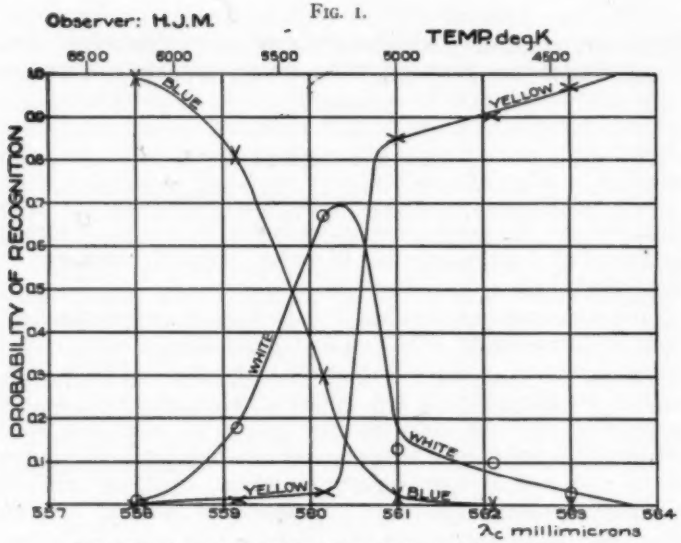
(1) To develop and test an experimental method of determining this hypothetical transition temperature, below which a Planckian radiator would appear yellow, at which it would appear white, and above which it would appear blue.

(2) To determine this temperature for a few individuals, under the following conditions:

(*a*) Field size, about 3.5°.
(*b*) Surrounding field dark. (No comparison field.)
(*c*) Intensity, medium; that is, low enough to avoid after images, and high enough to avoid the Purkinje effect.
(*d*) The effect of previous selective fatigue is eliminated.

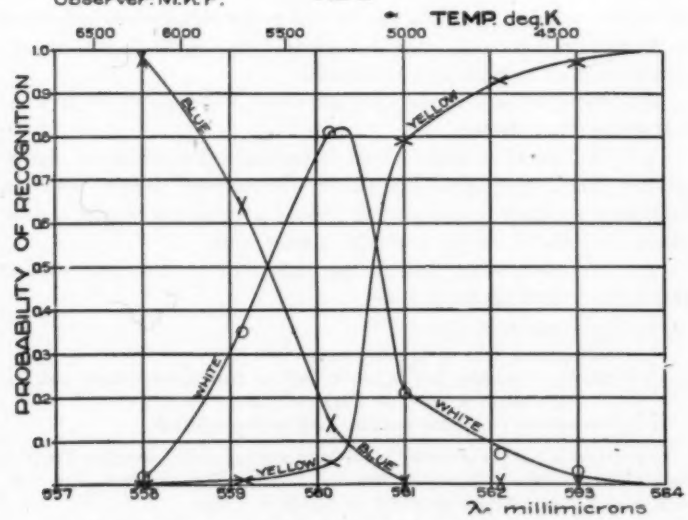
* Abstract of a paper presented at the joint meeting of the American Physical Society and the Optical Society of America, Chicago, Dec. 29, 1920. The complete paper will be published as a Bur. of Stands. Sci. Pap.

FIG. 1.



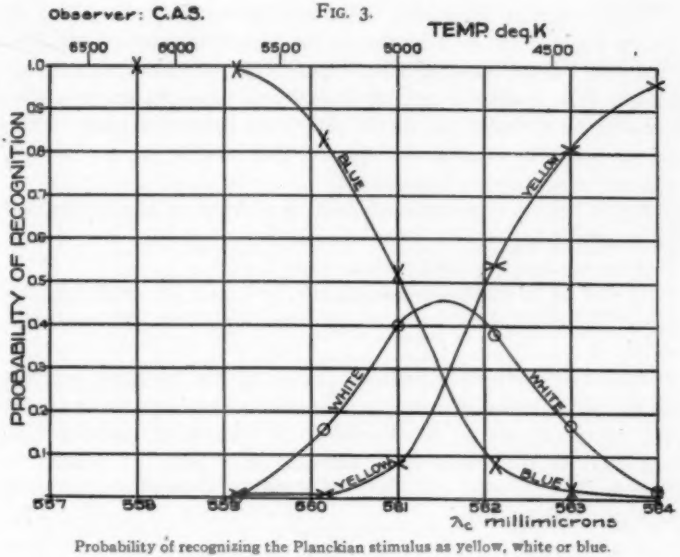
Probability of recognizing the Planckian stimulus as yellow, white or blue.

FIG. 2.



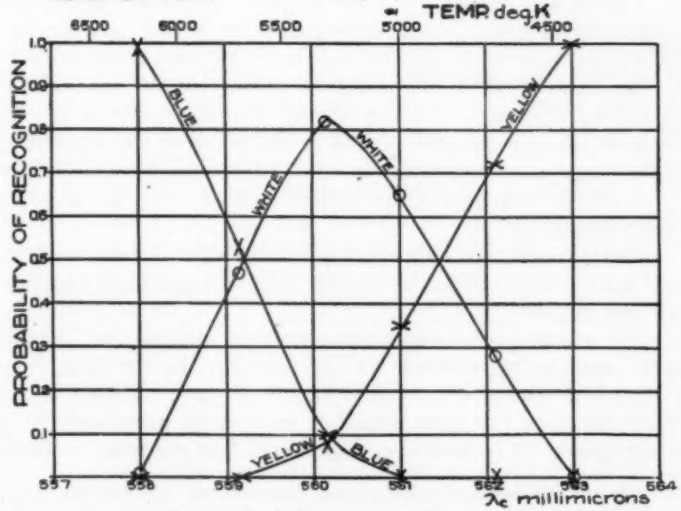
Probability of recognizing the Planckian stimulus as yellow, white or blue.

FIG. 3.



Probability of recognizing the Planckian stimulus as yellow, white or blue.

FIG. 4.



Probability of recognizing the Planckian stimulus as yellow, white or blue.

Experimental Methods.—The method of producing and adjusting the spectral distribution of the stimulus is this: Light from a lamp of known spectral distribution is modified by rotatory dispersion in a system of quartz plates and nicol prisms in such a way that by rotating one of the nicols the light emerging from it can be made to assume the spectral distribution of a Planckian radiator at any desired temperature between 4000° and 7000° K. Such a system is in effect a selective light filter of adjustable spectral transmission.[1]

Two methods of observation were used:

(1) The method of adjustment by trial, in which the observer, himself, adjusts the stimulus until he calls the sensation, "white."

(2) The method of answers, in which the operator conducting the experiment adjusts the stimulus to correspond to certain fixed temperatures of the hypothetical Planckian radiator; and then records the observer's reactions as "blue," "white," or "yellow," as the case may be. The latter method proved to be the more satisfactory.

Results and Conclusions.—Experimental results are given from four observers.

The results obtained by the method of answers are shown graphically in Figs. 1 to 4, inclusive. The stimuli are specified by the abscissæ, on which the scale of equal parts is in terms of λ_c, the wave-length of the centre of gravity of the spectral distribution of light.[2] Corresponding source temperatures are given in the upper margin. The ordinates show the probability of the observer's recognizing the stimulus as blue, white or yellow.

A graphic synopsis of the results is shown in Fig. 5, which is self-explanatory.

Previous selective fatigue has a notable effect on the temperature found, but recovery from such fatigue is rapid and complete in a few minutes.

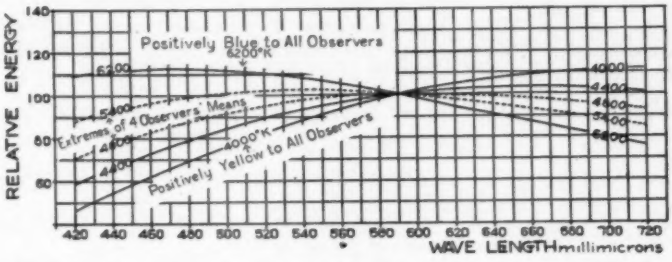
The average results of these observers indicate that "white light" may be represented: (1) *theoretically,* by the light from a Planckian radiator at a temperature of 5200° absolute; (2) *practically,* to a fair approximation, by average noon sunlight at Washington. It is, however, emphasized that the final establish-

[1] Cf. Priest: "Radiant Energy at High Temperatures," this JOURNAL, p. 178.

ment of such a standard should be based on a more extensive statistical investigation.

The results are compared with previously proposed standards for "average daylight." Accepting the results of this investigation tentatively as a standard for the stimulus of white, it is

FIG. 5.



Spectral Distributions of Radiant Energy Evoking the Sensations, White, Yellow and Blue.

Dotted curves, 5400° and 4800°K delineate the extreme range of four observers' final averages in determining the stimulus for white.

The solid curve, 6200°K shows the spectral distribution always recognized as blue by all four observers.

The solid curve, 4000°K shows the spectral distribution always recognized as yellow by all four observers.

The solid curve, 4400°K shows the spectral distribution always recognized as yellow by the three observers of ages less than 35 years.

observed that Ives' proposed "average daylight" (Planckian radiator at 5000° K) is a comparatively close approximation to this standard, while the sensation evoked by Gage's proposed "average daylight" (Planckian radiator at 7000° K) is undoubtedly blue.

An appendix to the paper sets forth the desirability of an extensive statistical determination and correlation of all the various relations between visual sensation and stimulus.

BUREAU OF STANDARDS,
January 29, 1921.

[3] See JOUR. OP. SOC. AM., 4, pp. 389-402; 1920.

NOTICES.

OPTICAL SOCIETY OF AMERICA, FIFTH ANNUAL MEETING.

The fifth annual meeting of the Optical Society of America was held in Chicago, at the University of Chicago, December 27, 28 and 29, 1920. On December 27th the dinner of the Society, attended by about forty members, was held at the Del Prado Hotel. The next meeting of the Optical Society will be in Rochester, N. Y., October, 1921. The following papers were presented before the Optical Society at the Fifth Meeting:

PROGRAM.

MONDAY MORNING AT 11 O'CLOCK.

(1) Courses in Optics and Optometry in Columbia University. JAMES P. C. SOUTHALL, Columbia University.

(2) Thermal Expansion of Wires Used in Glass Seals. C. G. PETERS and C. H. CRAGOE, Bureau of Standards.

(3) Refractive Index of Glass Through the Annealing Range. C. G. PETERS and C. H. CRAGOE, Bureau of Standards.

(4) Notes on the Theory of Photographic Spectrophotometers. E. D. TILLYER, American Optical Company.

(5) A New Ocular Micrometer. HERMANN KELLNER, Bausch & Lomb Optical Co.

(6) Presentation and Discussion of the Reports of the Committees on Nomenclature and Standards. P. G. NUTTING, General Chairman.

1. Colorimetry. L. T. Troland.
2. Lenses and Optical Instruments. J. P. C. Southall.
3. Optical Glasses. George W. Morey.
4. Photographic Materials. W. F. Meggers.
5. Photometry and Illumination. E. C. Crittenden.
6. Polarimetry. F. E. Wright.
7. Projection. L. A. Jones.
8. Pyrometry. W. E. Forsythe.
9. Reflectometry. A. H. Taylor.
10. Refractometry. C. A. Skinner.
11. Spectacle Lenses. E. D. Tillyer.
12. Spectrophotometry. A. H. Pfund.
13. Spectroradiometry. W. W. Coblentz.
14. Visual Sensitometry. Prentice Reeves.
15. Wave Lengths. W. F. Meggers.

About half of the above reports were presented before the general meeting by title only.

MONDAY AFTERNOON AT 2.30 O'CLOCK.

(7) A Comparison of Monochromatic Screens for Optical Pyrometry. W. E. FORSYTHE, Nela Research Laboratories.

(8) An Improved Form of Pickering Polarimeter for Gloss Measurements (by the Polarization Method). L. R. INGERSOLL, University of Wisconsin.

(9) An Unfamiliar Anomaly of Vision and Its Relation to Certain Optical Instruments. W. B. RAYTON, Bausch & Lomb Optical Co.

(10) Double Refraction of Glass Tubing as Indicating the Strains Present. A. Q. TOOL and C. G. EICHLIN, Bureau of Standards.

(11) Monocular and Binocular Perception of Contrast and Brightness. PRENTICE REEVES, Eastman Kodak Company.

(12) Systems of Color Standards. A. AMES, JR., Dartmouth College.

(13) A New Study of the Leucoscope and Its Application to Pyrometry. (Extension of work reported at N. Y., February, 1920.) IRWIN G. PRIEST, Bureau of Standards.

(14) Address of the Retiring President of the Optical Society of America. "Some Outstanding Problems of Physiological Optics." F. K. RICHTMYER, Cornell University.

(15) Atmospheric Corrections for the Harcourt Standard Pentane Lamp. E. B. ROSA, E. C. CRITTENDEN, A. H. TAYLOR, Bureau of Standards.

(16) Some Major Problems in Photometry. E. C. CRITTENDEN and J. F. SKOGLAND, Bureau of Standards.

(17) Comparative Tests as to the Accuracy of Various Methods for Precision Measurements of Focal Lengths (by title). W. O. LYTLE and A. K. BENNETT, Bureau of Standards.

TUESDAY MORNING AT 10 O'CLOCK.

(18) The Diffusion of Light in a Searchlight Beam (by title). ENOCH KARRER and U. M. SMITH, Bureau of Standards.

(19) Further Results on the Heat of Absorption of Glass. A. Q. TOOL and C. G. EICHLIN, Bureau of Standards.

(20) A Recent New System of Formulæ for Tracing Rays Through a Combination of Lenses. JAMES P. C. SOUTHALL, Columbia University.

(21) Notes on Lens Computation. HERMANN KELLNER, Bausch & Lomb Optical Co.

(22) A New Astronomical Lens. FRANK E. ROSS, Eastman Kodak Company.

(23) Note on the Extended Theory of the Sectored Disk Used in Photometry (by title). ENOCK KARRER, Bureau of Standards.

(24) Measurements of Aberrations of the Eye. C. A. PROCTOR and A. AMES, JR., Dartmouth College.

(25) Characteristics of Retinal Image. A. AMES, JR., and C. A. PROCTOR, Dartmouth College.

(26) Some Notes on Condenser Correction in Optical Projection (by title). G. W. MOFFIT, Eastman Kodak Company.

(27) The Use of the Ulbricht Sphere in Measuring Reflection and Transmission Factors (by title). ENOCH KARRER, Bureau of Standards.

(28) A Comparison of Retinoscopic, Subjective and Finally Acceptable Ocular Corrections. CHARLES SHEARD, American Optical Company.

(29) A New Method of Joining Glass. C. O. FAIRCHILD, Bureau of Standards.

(30) The Effect of Variations in Intensity of Illumination of Functions of Importance to the Working Eye (by title). C. E. FERREE and G. RAND, Bryn Mawr College.

(31) Optical Determination of Stress in Transparent Materials. A. L. KIMBALL, General Electric Co.

WEDNESDAY MORNING AT 9.30 O'CLOCK.

This was a joint session with the American Physical Society. The following papers were contributed by the Optical Society:

(32) Photographic Reproduction of Tone. L. A. JONES, Eastman Kodak Company.

(33) The Spectral Distribution of Energy Required to Evoke the Gray Sensation. IRWIN G. PRIEST, Bureau of Standards.

(34) The Propagation of Light in Rotating Systems. L. SILBERSTEIN, Eastman Kodak Company.

(Signed) PAUL D. FOOTE, *Sec.*, 1920.

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